

## Foundations Equations

Thermodynamics

$$\Delta L = \alpha L_0 \Delta T \quad (\text{linear thermal expansion})$$

$\Delta L$  - change in length (m)

$\alpha$  - linear expansion coefficient ( $K^{-1}$ )

$L_0$  - initial length (m)

$\Delta T$  - change in temperature (K)

$$\Delta V = \beta V_0 \Delta T \quad (\text{volume thermal expansion})$$

$\Delta V$  - change in volume ( $m^3$ )

$\beta$  - volume expansion coefficient ( $K^{-1}$ ) ( $\beta = 3\alpha$ )

$V_0$  - initial volume ( $m^3$ )

$$Q = nC\Delta\theta \quad (\text{molar heat capacity})$$

$Q$  - heat energy (J)

$n$  - no. of moles (mol)

$C$  - molar heat capacity ( $J K^{-1} mol^{-1}$ )

$\Delta\theta$  - change in temperature.

$$Q = mc\Delta\theta \quad (\text{specific heat capacity})$$

$m$  - mass (kg)

$c$  - specific heat capacity ( $J K^{-1} kg^{-1}$ )

$$Q = mL \quad (\text{specific latent heat})$$

$L$  - latent heat of fusion / vaporisation ( $J kg^{-1}$ )

$$\frac{dQ}{dt} = -kA \frac{dt}{dx} \quad (\text{rate of heat flow})$$

$k$  - constant of thermal conductivity ( $W m^{-1} K^{-1}$ )

$A$  - cross-sectional area ( $m^2$ )

$\frac{dt}{dx}$  - temperature gradient ( $K m^{-1}$ )

$$P = e\sigma AT^4 \quad (\text{Stefan-Boltzmann law of radiation})$$

$P$  - power (W)

$e$  - surface emissivity (ratio, no units)

$\sigma$  - Stefan-Boltzmann constant ( $5.67 \times 10^{-8} W m^{-2} K^{-4}$ )

$A$  - surface area ( $m^2$ )

$$pV = nRT \quad \text{or} \quad pV = Nk_B T \quad (\text{ideal gas equation})$$

$p$  = pressure (Pa)

$V$  = volume ( $\text{m}^3$ )

$n$  = number of moles (mol)

$R$  = molar gas constant ( $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ )

$T$  = temperature (K)

$k_B$  = Boltzmann constant ( $1.38 \times 10^{-23} \text{ J K}^{-1}$ )

$N$  = no. of molecules.

$$pV = \frac{1}{3} N m \bar{v}^2 \quad (\text{Kinetic theory})$$

$m$  = mass of molecule (kg)

$\bar{v}^2$  = root mean speed squared.

$$E_k = \frac{3}{2} k_B T \quad (\text{kinetic energy, average.})$$

$E_k$  = kinetic energy (J)

$$V_{\text{probable}} = \sqrt{\frac{2k_B T}{m}} \quad (\text{Most probable speed in a Boltzmann distribution})$$

$$V_{\text{average}} = \sqrt{\frac{9k_B T}{\pi m}} \quad (\text{Average speed in a Boltzmann distribution})$$

$$t_{\text{mean}} = \frac{V}{4\pi\sqrt{2}r^2 v_{\text{rms}} N} \quad (\text{Average time between collisions})$$

$r$  = radius of spherical molecule (m)

$N$  = no. of atoms

$v_{\text{rms}}$  = root mean squared speed ( $\text{ms}^{-1}$ )

$$\lambda = v_{\text{rms}} t_{\text{mean}} \quad (\text{mean 'free' path.})$$

$$\Delta U = \Delta Q - \Delta W \quad (1^{\text{st}} \text{ law of thermodynamics})$$

$\Delta U$  = change in internal energy (J)

$\Delta W$  = work done. (J)

$$W = \int_{v_1}^{v_2} p \, dV \quad (\text{Isothermal expansion of ideal gas.})$$

$p$  = pressure (Pa)

$$pV^\gamma = \text{constant} \quad (\text{Adiabatic expansion}) \quad \left(\gamma = \frac{C_p}{C_v}\right)$$

$$TV^{\gamma-1} = \text{constant} \quad (\text{Adiabatic expansion})$$

$$W = nRT \ln\left(\frac{v_2}{v_1}\right) \quad (\text{Isothermal expansion, work done.})$$

$$Q = nC_v \Delta T \quad (\text{Isochoric change in heat energy})$$

$$Q = nC_p \Delta T \quad (\text{Isobaric change in heat energy})$$



$$W = |Q_H| - |Q_C| \quad (\text{Work done for a heat engine})$$

$Q_H$  - heat supplied (J)

$Q_C$  - heat rejected (J)

$$E = \frac{W}{Q_H} \quad (\text{Efficiency of an engine})$$

$$= 1 - \frac{|Q_C|}{|Q_H|}$$

$$K = \frac{|Q_C|}{W} \quad (\text{Performance coefficient for a heat pump.})$$

$$W = \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1) = -n C_V \Delta T \quad (\text{work done in an adiabatic expansion})$$

$$E = 1 - \frac{T_C}{T_H} \quad (\text{Efficiency of the Carnot cycle.})$$

$T_C$  - temperature of cold reservoir (K)

$T_H$  - temperature of hot reservoir (K)

$$\Delta S = \frac{Q}{T} \quad (\text{Entropy change for a reversible process.})$$

$\Delta S$  - change in entropy, ( $\text{JK}^{-1}$ )

$$\Delta S = \int_{\text{state 1}}^{\text{state 2}} \frac{dQ}{T} \quad (\text{Entropy change for non-isothermal})$$

## Waves

$$u(x, t) = A \cos(kx - \omega t) \quad (\text{general waveform, sinusoidal.})$$

$A$  - amplitude (m)

$x$  - displacement (m)

$t$  - time (s)

$\omega$  - angular velocity / frequency ( $\text{rads}^{-1}$ )

$k$  - wave number ( $\text{m}^{-1}$ )

$$k = \frac{2\pi}{\lambda} \quad (\text{wavenumber})$$

$\lambda$  - wavelength (m)

$$\omega = 2\pi f \quad (\text{angular velocity})$$

$f$  - frequency (Hz)

$$c = f\lambda \quad (\text{wavespeed equation})$$

$c$  = wavespeed ( $\text{ms}^{-1}$ )

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (\text{wave equation})$$

$c$  - wavespeed ( $\text{ms}^{-1}$ )

$$v_g = \frac{d\omega}{dk} \quad (\text{group velocity})$$

$v_g$  - group velocity ( $\text{ms}^{-1}$ )

$\omega$  - angular velocity ( $\text{rad s}^{-1}$ )

$k$  - wavenumber ( $\text{m}^{-1}$ )

$$c = \frac{\omega}{k} \quad (\text{wavespeed})$$

$\omega$  - angular velocity ( $\text{rad s}^{-1}$ )

$k$  - wave number ( $\text{m}^{-1}$ )

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad (\text{speed of light})$$

$\mu_0$  - permeability of free space

$\epsilon_0$  - permittivity of free space

$$\omega(k) = k \sqrt{\frac{B}{\rho}} \quad (\text{dispersion relation})$$

$B$  - Bulk modulus of air ( $\text{Pa}$ )

$\rho$  - density of air ( $\text{kg m}^{-3}$ )

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\omega}{k} = v \quad (\text{group velocity in air})$$

\*  $v_g$  - group velocity ( $\text{ms}^{-1}$ )

$v$  - wavespeed ( $\text{ms}^{-1}$ )

$$\mu \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} \quad (\text{wave equation for mechanical waves})$$

$\mu$  - mass per unit length ( $\text{kg m}^{-1}$ )

$T$  - tension ( $\text{N}$ )

$$c = \sqrt{\frac{T}{\mu}} \quad (\text{wavesped of mechanical waves})$$



$$P = -T \frac{dy}{dx} \cdot \frac{dy}{dt} \quad (\text{power transfer of mech. wave.})$$

T - tension (Pa)

eg: consider a sinusoidal wave,  $y(t, x) = A \cos(kx - \omega t)$

$$P = T A^2 k \omega \sin^2(kx - \omega t)$$

$$k = \omega \sqrt{\frac{\mu}{T}} \quad (\text{equating wavespeed equations})$$

$$\text{Then, } P = \omega^2 A^2 \sqrt{\mu T} \sin^2(kx - \omega t)$$

$$Z = \sqrt{\mu T} \quad (\text{impedance})$$

Z - impedance ( $\text{kg s}^{-1}$ )

$\mu$  - mass per unit length ( $\text{kg m}^{-1}$ )

T - tension (N)

$$P_{\text{average}} = \frac{1}{2} \omega^2 A^2 Z \quad (\text{average power transmitted})$$

$\omega$  - angular velocity ( $\text{rads}^{-1}$ )

A - amplitude (m)

Z - impedance ( $\text{kg s}^{-1}$ )

$$\text{Strain} = \frac{\Delta \sigma}{\Delta x}$$

$$\text{Stress} = \frac{F}{A}$$

$$\text{Young's modulus} = Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F \Delta x}{A \Delta \sigma}$$

$$v = \sqrt{\frac{Y}{\rho}} \quad (\text{wavespeed of longitudinal elastic waves along a rod.})$$

Y - young's modulus of rod (Pa)

$\rho$  - density of rod

$$v = \sqrt{\frac{G}{\rho}} \quad (\text{wavespeed of a shear wave})$$

G - shear modulus of rod (Pa)

$$P = A k_B \sin(kx - \omega t) \quad (\text{pressure of wave in Bulk gas/liquid})$$

$k_B$  - Boltzmann constant

$$v = \sqrt{\frac{B}{\rho}} \quad (\text{wavespeed in liquids and gases})$$

B - Bulk modulus (Pa)

$\rho$  - density ( $\text{kg m}^{-3}$ )

$$v_{\text{sound}} = \sqrt{\frac{\gamma p v}{Nm}} = \sqrt{\frac{\gamma k_B T}{m}} \quad (\text{speed of sound in ideal gas})$$

$\gamma$  - adiabatic index

T - temperature (K)

m = mass of one molecule (kg)

$$v_{\text{sound}} = v_{\text{rms}} \sqrt{\frac{\gamma}{3}} \quad (\text{using ideal gas relations})$$

$v_{\text{rms}}$  - root mean speed squared ( $\text{ms}^{-1}$ )

$$v_p = \sqrt{\frac{B + \frac{4}{3}\rho v^2}{\rho}} \quad (p\text{-wavespeed, compressional waves})$$

B - Bulk modulus (Pa)

$$v_s = \sqrt{\frac{G}{\rho}} \quad (s\text{-wavespeed, shear waves})$$

G - shear modulus (Pa)

$\rho$  - density ( $\text{kg m}^{-3}$ )

$$T = \frac{A_t}{A_i} = \frac{Z_t}{Z_i + Z_t} \quad (\text{transmission coefficient})$$

$A_t$  - amplitude of transmitted wave (m)

$A_i$  - amplitude of incident wave (m)

$Z_i$  - impedance of incident wave ( $\text{kg s}^{-1}$ )

$Z_t$  - impedance of transmitted wave ( $\text{kg s}^{-1}$ )

$$R = \frac{A_r}{A_i} = \frac{Z_i - Z_t}{Z_i + Z_t} \quad (\text{reflection coefficient})$$

$A_r$  - amplitude of reflected wave (m)

$$Z_{\text{fluid}} = \sqrt{\rho B} \quad (\text{impedance of a fluid})$$

$\rho$  - density of fluid ( $\text{kg m}^{-3}$ )

B - bulk modulus



$$\lambda = \frac{2L}{n} \quad (\text{wavelength of standing wave of fixed ends})$$

$n$  - harmonic number (integer)

$L$  - length between fixed ends (m)

$$c = f\lambda \quad (\text{wave equation})$$

$c$  - wavespeed ( $\text{ms}^{-1}$ )

$f$  - frequency (Hz)

$\lambda$  - wavelength (m)

$$f' = f \left( \frac{c + u_o}{c + u_s} \right) \quad (\text{Doppler effect for sound})$$

$f'$  - observed frequency (Hz)

$f$  - emitted frequency (Hz)

$c$  - speed of sound ( $\text{ms}^{-1}$ )

$u_o$  - speed of observer ( $\text{ms}^{-1}$ )

$u_s$  - speed of source ( $\text{ms}^{-1}$ )

$$f' = f \sqrt{\frac{c - v}{c + v}} \quad (\text{Doppler effect for em. waves})$$

$c$  - speed of light ( $\text{ms}^{-1}$ )

$v$  - velocity of observer ( $\text{ms}^{-1}$ )

$$I = I_0 \cos^2 \theta \quad (\text{Malus's law})$$

$I$  - intensity ( $\text{Wm}^{-2}$ )

$I_0$  - initial intensity ( $\text{Wm}^{-2}$ )

$\theta$  - angle of polariser ( $^\circ$ )

$$I = \frac{1}{2} \epsilon_0 c E_0^2 \quad (\text{Intensity of em. wave})$$

$\epsilon_0$  - permittivity of free space

$c$  - speed of light ( $\text{ms}^{-1}$ )

$E_0$  - amplitude (m)

$$\frac{dp}{dt} = \frac{I}{c} \quad (\text{light pressure})$$

$I$  - intensity ( $\text{Wm}^{-2}$ )

$c$  - speed of light ( $\text{ms}^{-1}$ )

$$n = \frac{c}{v} \quad (\text{refractive index})$$

$n$  - refractive index (ratio) ( $n > 1$ )

$c$  - speed of light in vacuum ( $\text{ms}^{-1}$ )

$v$  - speed of light in material ( $\text{ms}^{-1}$ )

$$n_i \sin \theta_i = n_t \sin \theta_t \quad (\text{Snell's law})$$

$n_i$  - incident refractive index (ratio)

$n_t$  - transmitted refractive index (ratio)

$\theta_i$  - angle of incidence ( $^\circ$ )

$\theta_t$  - angle of refraction ( $^\circ$ )

$$T = \frac{4 n_i n_t}{(n_i + n_t)^2} \quad (\text{transmission coefficients for refraction})$$

$$R = \frac{(n_i - n_t)^2}{(n_i + n_t)^2} \quad (\text{reflection coefficients for refraction})$$

$$\tan \theta_p = \frac{n_t}{n_i} \quad (\text{polarization angle})$$

$\theta_p$  - polarization angle ( $^\circ$ )

$$n \lambda = d \sin \theta \quad (\text{constructive interference})$$

$n$  - integer, peak number

$\lambda$  - wavelength (m)

$d$  - source separation (m)

$\theta$  - angle ( $^\circ$ )

$$I_{\text{total}} = I_0 \cos^2 \left( \frac{k d}{2} \sin \theta \right) \quad (\text{total intensity})$$

$k$  - wavenumber ( $\text{m}^{-1}$ )

$$I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \quad (\text{diffraction from one slit})$$

$$\beta = \frac{k a}{2} \sin \theta$$

$I_0$  - initial intensity ( $\text{Wm}^{-2}$ )

$a$  - slit separation (m)



$$I(\theta) = I_0 \cos^2 \left( \frac{kd \sin \theta}{2} \right) \left( \frac{\sin \beta}{\beta} \right)^2 \quad (\text{diffraction from 2 identical slits})$$

$$I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin \left( N \frac{kd}{2} \sin \theta \right)}{\sin \left( \frac{kd}{2} \sin \theta \right)} \right)^2 \quad (\text{diffraction from } N \text{ identical slits})$$

~~A~~  $N$  - number of slits

$$\sin \theta = \frac{1.22 \lambda}{D} \quad (\text{circular aperture diffraction})$$

$D$  - diameter of circular aperture (m)

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad (\text{first harmonic frequency})$$

$L$  - length of string (m)

$T$  - tension (N)

$\mu$  - mass per unit length ( $\text{kg m}^{-1}$ )

$$w = \frac{\lambda D}{s} \quad (\text{double-slit formula})$$

$w$  - fringe spacing (m)

$D$  - distance from screen to slits (m)

$s$  - slit separation (m)

### Mechanics equations

Px148

$$\vec{p} = m\vec{v} \quad (\text{momentum})$$

$m$  - mass (kg)

$\vec{v}$  - velocity ( $\text{ms}^{-1}$ )

$\vec{p}$  - momentum ( $\text{kg ms}^{-1}$ )

$$\vec{F} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a} \quad (\text{Newton's 2nd law})$$

← (only holds for constant mass)

$\vec{a}$  - acceleration ( $\text{ms}^{-2}$ )

$$\vec{F}_s \leq \mu_s \vec{N} \quad (\text{static friction})$$

$\vec{F}_s$  - frictional force (N)

$\vec{N}$  - normal reaction force (N)

$\mu_s$  - coefficient of static friction

•  $F_k = \mu_k N$  (kinetic friction)

$F_k$  - kinetic friction force (N)

$\mu_k$  - coefficient of kinetic friction

[In general,  $\mu_k < \mu_s$ ]

•  $\vec{F}_{12} = - \frac{G m_1 m_2}{|\vec{r}|^2} \hat{r}$  and  $|\vec{F}| = \frac{G m_1 m_2}{r^2}$  (Newton's law of gravitation)

$\vec{F}_{12}$  - force of 1 on 2 (N)

$G$  - gravitational constant

$m_1$  - mass 1 (kg)

$m_2$  - mass 2 (kg)

$\hat{r}$  - unit vector pointing from 1 to 2 (m)

•  $\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$  (position of centre of mass)

•  $\vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm}$  (position of centre of mass of continuous bodies.)

•  $\int \vec{r} dm = \left[ \int x dm, \int y dm, \int z dm \right]$

•  $v = u + at$

•  $s = ut + \frac{1}{2} at^2$

•  $s = vt - \frac{1}{2} at^2$  (SUVAT eqns)

•  $v^2 = u^2 + 2as$

•  $s = \left( \frac{u+v}{2} \right) t$

$v$  - final velocity ( $\text{ms}^{-1}$ )

$u$  - initial velocity ( $\text{ms}^{-1}$ )

$s$  - displacement (m)

$a$  - acceleration ( $\text{ms}^{-2}$ )

$t$  - time (s)



- $v - u = \int_0^t a(t) dt$  (time-dependent acceleration)
- $\frac{1}{2}(v^2 - u^2) = \int_0^x a(x) dx$  (position-dependent acceleration)
- $\int_u^v \frac{1}{a(v)} dv = t$  (velocity-dependent acceleration)
- $T = \frac{1}{2}mv^2$  (kinetic energy)
- $W = \int F dx = \Delta T$  (work done)

T - kinetic energy (J)

F - force (N)

W - work done (J)

- $\oint \vec{F} \cdot d\vec{r} = 0$  (work done in a loop)

- $E = T + U$  (conservation of energy)

E - total energy (J)

U - potential energy (J)

- $F = -\frac{dU}{dx}$  (force from potential)

- $\vec{F} = -\frac{\partial u}{\partial x} \hat{i} - \frac{\partial u}{\partial y} \hat{j} - \frac{\partial u}{\partial z} \hat{k} = -\vec{\nabla} u$  (vector force from potential)

- $u(r) = -\frac{GMm}{r}$  (gravitational potential energy)

G - gravitational constant

M - large mass (kg)

m - small mass (kg)

r - distance between masses (m)

- $v \geq \sqrt{\frac{2GM}{R}}$  (escape velocity)

M - mass of planet (kg)

R - radius of planet (m)

- $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$  (dot product) (Power)

$P$  - power (W)

$\vec{F}$  - force (N)

$\vec{v}$  - velocity ( $\text{ms}^{-1}$ )

- $F = -kx$  (force from spring)

$k$  - spring constant ( $\text{Nm}^{-1}$ )

$x$  - extension (m)

- $\frac{d^2x}{dt^2} + \omega^2 x = 0$  (equation of SHM)

- $\omega^2 = \frac{k}{m}$  (angular velocity)

- $x = A \cos(\omega t) + B \sin(\omega t)$  (solution to SHM eq.)

$A$  and  $B$  - constants

- $f = \frac{\omega}{2\pi} = \frac{1}{T}$
- $T = \frac{2\pi}{\omega} = \frac{1}{f}$

(period and frequency relations)

- $x = A' \cos(\omega t + \phi)$  (alternative solution)

$A'$  - amplitude (m)

$\phi$  - phase angle (rad.)

- $T = 2\pi \sqrt{\frac{m}{k}}$  (period of mass-spring system)

- $U = \frac{1}{2} kx^2$  (potential energy of a spring)

- $U_{av} = \frac{1}{4} kA^2$  (average energy of an oscillating spring)

- $\ddot{x} + \gamma \dot{x} + \omega^2 x = 0$  (damped SHM eq.)

- $\gamma = \frac{b}{m}$  (damping coefficient)

$b$  - damping constant

- $p = -\frac{\gamma}{2} \pm \sqrt{\left(\frac{\gamma}{2}\right)^2 - \omega^2}$  (solution to SHM damped eq.)



- $\gamma < 2\omega$  (light / under-damping)
- $\gamma > 2\omega$  (heavy / over-damping)
- $\gamma = 2\omega$  (critical damping)
- $x = |a| e^{-\gamma t/2} \cos(\omega' t + \phi)$  (light damping solution)
- $\omega' = \sqrt{\omega^2 - \left(\frac{\gamma}{2}\right)^2}$  (reduced angular velocity)

$\phi$  - phase angle.

$a$  - constant

- $z = a e^{p_1 t} + b e^{p_2 t}$  (over-damped solution)
- $x = \text{Re}(z)$

$p_1$  and  $p_2$  are solutions of the damped eqn.

- $z = (a + bt) e^{-\gamma t/2}$  (critically-damped solution)
- $x = \text{Re}(z)$

$a$  and  $b$  are constants.

- $\ddot{z} + \gamma \dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i\omega t}$  (forced oscillations)

$\omega_0$  - natural frequency ( $s^{-1}$ )

$\omega$  - applied frequency ( $s^{-1}$ )

$F_0$  - force applied (N)

- $a = \frac{F_0/m}{(\omega_0^2 - \omega^2) + i\gamma\omega}$  (solution to forced oscillations)

- $|a| = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$  (amplitude modulus)

- $\tan \phi = -\frac{\gamma\omega}{\omega_0^2 - \omega^2}$  (phase angle)

- $|a|_0 = \frac{F_0}{m\omega_0^2}$  (for  $\omega = 0$ )

- $\frac{d^2\theta}{dt^2} = -\frac{g}{L} \sin \theta$

$\theta$  - angle from vertical (rad)

$g$  - gravity ( $ms^{-2}$ )

$l$  - length of string (m)

•  $\omega^2 = \frac{g}{l}$  (frequency of pendulum)

•  $T = 2\pi \sqrt{\frac{l}{g}}$  (period of pendulum)

•  $m dv = -v_e dm$  (infinitesimal momentum conservation for rockets)

•  $v - v_0 = v_e \ln \left( \frac{M_0}{m} \right)$  (rocket eqn.)

$v_0$  - initial velocity ( $\text{ms}^{-1}$ )

$v$  - final velocity ( $\text{ms}^{-1}$ )

$v_e$  - exhaust velocity ( $\text{ms}^{-1}$ )

$M_0$  - initial mass (kg)

$m$  - final mass (kg)

•  $m \frac{dv}{dt} = -v_e \frac{dm}{dt}$  (rocket thrust)

•  $\omega = \frac{v}{r} = \frac{d\theta}{dt}$  (angular velocity)

•  $\vec{v} = \vec{\omega} \times \vec{r}$  (angular velocity vector)

$\vec{v}$  - velocity ( $\text{ms}^{-1}$ )

$\vec{r}$  - position vector (m)

•  $a = \omega v = \omega^2 r = \frac{v^2}{r}$  (centripetal acceleration)

•  $v = \sqrt{\frac{GM_p}{R}}$  (orbital velocity)

$v$  - orbital velocity ( $\text{ms}^{-1}$ )

$G$  - gravitational constant  $\text{N m}^2 \text{kg}^{-2}$

$M_p$  - mass of planet/star (kg)

$R$  - orbital radius (m)

•  $\omega^2 = \frac{4\pi^2}{P^2} = \frac{GM_p}{R^3}$  (Kepler's 3<sup>rd</sup> law)

$P$  - orbital period (s)



$$\vec{\tau} = \vec{r} \times \vec{F} \quad (\text{torque})$$

$\vec{\tau}$  - torque (Nm)

$\vec{F}$  - force applied (N)

$\vec{r}$  - position vector (m)

$$m = Fd \quad (\text{moments})$$

$m$  - moment (Nm)

$$\vec{L} = \vec{r} \times \vec{p} \quad (\text{angular momentum})$$

$\vec{L}$  - angular momentum ( $\text{kg m}^2 \text{s}^{-1}$ )

$\vec{r}$  - position vector (m)

$\vec{p}$  - linear momentum ( $\text{kg ms}^{-1}$ )

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad (\text{torque + angular momentum relation})$$

$$I = mr^2 \quad (\text{moment of inertia})$$

$I$  - moment of inertia ( $\text{kg m}^2$ )

$$L = I\omega \quad (\text{angular momentum of particle moving w/ circular motion})$$

$$T = \frac{1}{2} I \omega^2 \quad (\text{kinetic energy "})$$

$$I = \int r^2 dm = \int r^2 \rho dV \quad (\text{moment of inertia for rigid bodies})$$

$r$  - perpendicular distance from axis to element (m)

$\rho$  - density ( $\text{kg m}^{-3}$ )

$$I = MR^2 \quad (\text{moment of inertia for a thin ring})$$

$M$  - mass of ring (kg)

$R$  - radius of ring (m)

$$I = \frac{1}{2} MR^2 \quad (\text{moment of inertia for a circular disc})$$

$M$  - mass of disc (kg)

$R$  - radius of disc (m)

$$I = \frac{2}{5} MR^2 \quad (\text{moment of inertia for uniform sphere})$$

$M$  - mass of sphere (kg)

$R$  - radius of sphere (m)

$$I = \frac{1}{12} ML^2 \quad (\text{moment of inertia for a thin rod, to axis through its centre.})$$

$m$  - mass of rod (kg)

$L$  - length of rod (m)

•  $I = \frac{1}{3} mL^2$  (same rod, but  $\perp$  axis through one end)

•  $I$  (any axis) =  $I_{cm} (||) + Md^2$  (parallel axis theorem)

$I_{cm} (||)$  = moment of inertia of centre of mass around a parallel axis (kgm<sup>2</sup>)

•  $\omega^2 = \frac{4\pi^2}{P^2} = \frac{1}{1 + (I_{cm}/Md^2)} \left(\frac{g}{d}\right)$  (compound pendulum)

$P$  - period of oscillation (s)

$d$  - distance from pivot (m)

•  $\frac{dL}{dt} = 0$  (conservation of angular momentum)

(only if no external torque applied)

•  $T = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2$  (angular work)

$v_{cm}$  - velocity of centre of mass (ms<sup>-1</sup>)

$I$  - moment of inertia around an axis through centre of mass parallel to  $\omega$ . (kgm<sup>2</sup>)

•  $P = \frac{dW}{dt} = T\omega = \vec{T} \cdot \vec{\omega}$  (angular power)

•  $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$  (centre of mass velocity)

$m_1$  - mass of 1<sup>st</sup> object (kg)

$m_2$  - mass of 2<sup>nd</sup> object (kg)

$v_1$  - velocity of 1<sup>st</sup> object (ms<sup>-1</sup>)

$v_2$  - velocity of 2<sup>nd</sup> object (ms<sup>-1</sup>)

•  $u_1 = v_{cm} + u_1'$   
•  $u_2 = v_{cm} + u_2'$  } (relative velocities)

•  $v_1' = v_2' = 0$  (completely inelastic collisions)

• primed velocities are in c.o.m. frame.



## Special Relativity

PX148

## Galilean transforms:

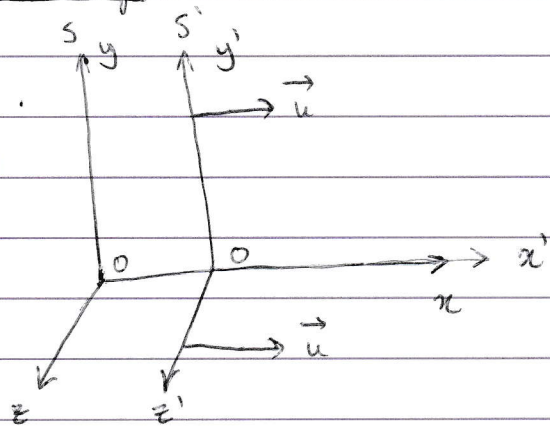
$$x' = x - ut \quad x = x' + ut$$

$$y' = y \quad y = y'$$

$$z' = z \quad z = z'$$

$$t' = t \quad t = t'$$

for 2 frames,  $S$  and  $S'$   
 while  $S'$  moves with velocity  $u \hat{i} \text{ ms}^{-1}$ .



## Lorentz transforms:

$$x' = \gamma (x - ut)$$

$$x = \gamma (x' + ut')$$

$$y' = y$$

$$y = y'$$

$$z' = z$$

$$z = z'$$

$$t' = \gamma \left( t - \frac{ux}{c^2} \right)$$

$$t = \gamma \left( t' + \frac{ux'}{c^2} \right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \quad (\text{Lorentz factor})$$

Note: to find the inverse transforms, set  $u$  to  $-u$ , and swap  $t \rightarrow t'$ ,  $x \rightarrow x'$ , etc.

$$L = \frac{L_0}{\gamma} \quad (\text{length contraction})$$

$L$  - observed length (m)

$L_0$  - rest length (m)

$\gamma$  - Lorentz factor

$$\Delta t = \gamma \Delta t_0 \quad (\text{time dilation})$$

$\Delta t$  - observed time (s)

$\Delta t_0$  - 'proper' time (s)

$$f = \frac{1}{\gamma \left( 1 + \frac{u}{c} \right)} \quad (\text{relativistic doppler shift, moving away})$$

$u$  - speed of light moving away ( $u < c$ ) ( $\text{ms}^{-1}$ )

$f_0$  - 'proper' frequency (Hz)

$f$  - observed frequency (Hz)

$\gamma$  - Lorentz factor.

$$\frac{f}{f_0} = \frac{1}{\gamma \left(1 - \frac{u}{c}\right)} \quad (\text{relativistic doppler shift, moving towards})$$

$$\frac{\lambda}{\lambda_0} = \gamma \left(1 \pm \frac{u}{c}\right) \quad (\text{wavelengths instead})$$

$$\left. \begin{aligned} v &= \frac{v' + u}{1 + \frac{uv'}{c^2}} \\ v' &= \frac{v - u}{1 - \frac{uv}{c^2}} \end{aligned} \right\} \begin{array}{l} \text{(relativistic addition} \\ \text{of velocities)} \end{array}$$

The other components of velocity can be calculated using these eqns.

$$m = \gamma m_0 \quad (\text{relativistic mass})$$

$m_0$  - rest mass (kg)

$m$  - observed mass (kg)

$$\vec{p} = \gamma m_0 \vec{v} \quad (\text{relativistic momentum})$$

$\vec{v}$  - velocity ( $\text{ms}^{-1}$ )

$\vec{p}$  - relativistic momentum ( $\text{kg ms}^{-1}$ )

$$\vec{F} = \frac{d\vec{p}}{dt} = \gamma m_0 \vec{a} + \frac{d\gamma}{dt} m_0 \vec{v} \quad (\text{relativistic force, uncommon though.})$$

$$E_k = (\gamma - 1) m_0 c^2 \quad (\text{relativistic kinetic energy})$$

$E_k$  - kinetic energy (J)

$$E = E_0 + E_k = \gamma m_0 c^2 = mc^2 \quad (\text{total energy})$$

$E$  - total energy (J)

$E_0$  - rest mass energy (J)

$m_0$  - rest mass (kg)

$m$  - observed mass (kg)

$$E^2 - p^2 c^2 = m_0^2 c^4 = E_0^2 \quad (\text{energy-momentum relation, Lorentz invariant})$$



Particle Physics

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad (\text{de Broglie wavelength})$$

$\lambda$  - de Broglie wavelength (m)

$h$  - Planck's constant

$m$  - mass of particle (kg)

$v$  - velocity of particle ( $\text{ms}^{-1}$ )

$p$  - momentum of particle ( $\text{kgms}^{-1}$ )

$$E = mc^2 \quad (\text{energy equivalent for mass})$$

$$E = pc \quad (\text{energy equivalent for momentum})$$

$$E^2 = p^2 c^2 + m^2 c^4 \quad (\text{Total energy})$$

$$M_{\text{initial}} \geq \sum M_{\text{final}} \quad (\text{decay})$$

$$L = L_e + L_\mu + L_\tau \quad (\text{Global lepton number})$$

$L_e$  - electron number

$L_\mu$  - muon number

$L_\tau$  - tau number

$$F = \alpha_{\text{em}} \frac{q^2}{r^2} \quad (\text{Electromagnetic force})$$

$\alpha_{\text{em}}$  - intrinsic strength of force

$q$  - charge (C)

$r$  - distance (m)

$$F = \frac{\alpha_s}{r^2} - k \quad (\text{Strong force})$$

$\alpha_s$  - strong force coupling constant

$k$  - constant ( $\approx 1.6 \text{ eV fm}^{-1}$ )

$$F = \alpha_w \frac{e^{-\frac{r}{R}}}{r^2} \quad (\text{Weak force})$$

$r$  - distance

$\alpha_w$  - intrinsic strength

$$R = \frac{\hbar}{m_{W,Z} c}$$

$m$  - mass of boson (kg)

$$\hbar = \frac{h}{2\pi} \quad (\text{reduced Planck constant})$$

$h$  - Planck constant

$$N(t) = N_0 e^{-\Gamma t} = N_0 e^{-\frac{t}{\tau}} \quad (\text{Decay eqn.})$$

$\Gamma$  - decay rate ( $s^{-1}$ )

$N_0$  - initial number of particles

$t$  - time (s)

$\tau$  - decay time of particles (s)

$$BR_i = \frac{\Gamma_i}{\Gamma} = \frac{\Gamma_i}{\sum_j \Gamma_j} \quad (\text{Branching ratio equation})$$

$\Gamma_i$  - decay rate for mode  $i$ . ( $s^{-1}$ )

$\Gamma$  - total decay rate ( $s^{-1}$ )

$BR_i$  - Branching ratio of  $i$ .

$$s = (E_1 + E_2)^2 - |p_1 + p_2|^2 c^2 \quad (\text{Mandelstam - } s \text{ invariant})$$

$E_1$  - energy of 1<sup>st</sup> particle (J)

$E_2$  - energy of 2<sup>nd</sup> particle (J)

$p_1$  - momentum of 1<sup>st</sup> particle ( $kgms^{-1}$ )

$p_2$  - momentum of 2<sup>nd</sup> particle ( $kgms^{-1}$ )

$$s_{\text{com}} = (E_1 + E_2)^2 \quad (\text{Mandelstam - } s \text{ in centre of mass frame})$$

$$s_{\text{fixed}} = 2E_1 m_2 c^2 \quad (\text{Mandelstam - } s \text{ in fixed target frame if body 2 is at rest.})$$

$$r = \frac{mv}{qB} = \frac{p}{qB} \quad (\text{particles (charged) moving in a magnetic field})$$

$m$  - mass of particle (kg)

$v$  - linear velocity ( $ms^{-1}$ )

$B$  - magnitude of magnetic field (T)

$q$  - magnitude of electric charge (C)

$p$  - momentum ( $kgms^{-1}$ )

$r$  - radius of path (m)

$$t = \frac{\pi m}{qB} \quad (\text{time spent in one dee of cyclotron})$$



$$r = \frac{\gamma m v}{2B} \quad (\text{relativistic radius})$$

$\gamma$  - Lorentz factor

$$f = \frac{qB}{2\pi\gamma m_0} \quad (\text{cyclotron frequency})$$

$$P(x) = e^{-\frac{x}{x_0}} \quad (\text{radiation probability}) \quad (\text{photons})$$

$P(x)$  - probability of an electron or photon passing unaffected

$x$  - thickness of material (m)

$x_0$  - radiation length (m)

$$P(x) = e^{-\frac{x}{\lambda_1}} \quad (\text{radiation probability}) \quad (\text{hadrons})$$

$\lambda_1$  - hadronic interaction length (m)

## Electricity and Magnetism

PK120

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0} \cdot \frac{\hat{r}}{r^2} \quad (\text{Coulomb's law})$$

$\vec{F}_{12}$  - force of 1 on 2 (N)

$q_1$  - charge 1 (C)

$q_2$  - charge 2 (C)

$\hat{r}$  - unit vector from 1 to 2 (m)

$\epsilon_0$  - permittivity of free space.

$$\vec{F}_0 = \sum_{i=0}^N \vec{F}_{i0} = \frac{q_0}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i (\vec{r}_0 - \vec{r}_i)}{|\vec{r}_0 - \vec{r}_i|^3} \quad (\text{principle of superposition})$$

just the vector sum of all forces acting on  $i$ .

$$\vec{E}(\vec{r}) = \lim_{q_0 \rightarrow 0} \frac{\vec{F}_0}{q_0} \quad (\text{Electric field relation})$$

$\vec{E}$  - electric field strength ( $\text{NC}^{-1}$  or  $\text{Vm}^{-1}$ )

$q_0$  - test charge (C)

$\vec{F}_0$  - force of charge (N)

$$\vec{F}_0 = q_0 \vec{E}(\vec{r}_0) \quad (\text{force on charge in electric field.})$$

$\vec{r}_0$  - position of charge (m)

- $\vec{E}(r) = \frac{q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$  (electric field around a point charge.)

this is assuming it is at the origin.

- $\lambda = \frac{dq}{dl}$  (linear charge density)

- $\sigma = \frac{dq}{ds}$  (surface charge density)

- $\rho = \frac{dq}{dv}$  (volume charge density)

- $\vec{p} = q\vec{d}$  (electric dipole moment)

$\vec{p}$  - electric dipole moment (Cm)

$\vec{d}$  - distance between two poles, pointing from negative to pos. (m)

- $\vec{\tau} = \vec{p} \times \vec{E}$  (torque from a dipole)

$\vec{\tau}$  - torque about centre of dipole (Nm)

- $u = -\vec{p} \cdot \vec{E}$  (potential energy)

dependant on  $\phi$ , since  $u = -\vec{p} \cdot \vec{E} = -pE \cos \phi$

- $\vec{E} \approx \frac{1}{4\pi\epsilon_0 r^3} [3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}]$  (electric field from a dipole.)

$r$  - position from midpoint of dipole (m)

$\vec{p}$  - dipole moment (Cm)

- $\vec{E} \cdot \vec{S} = \Phi_E$  (electric flux)

$\vec{S}$  - surface area (m<sup>2</sup>)

$\Phi_E$  - electric flux (Vm)

- $\Phi_E = \iint_S \vec{E} \cdot d\vec{S}$  (for an arbitrary surface.)

this is a surface integral.

- $\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc.}}}{\epsilon_0}$  (Gauss's law)

$q_{\text{enc.}}$  - enclosed charge by gaussian surface (C)

- $\vec{E} = 0$  (electric field INSIDE conductors)

for conductors only.



$$W_{A \rightarrow B} = \int_a^b \vec{F} \cdot d\vec{l} \quad (\text{work done in force field})$$

$$W_{A \rightarrow B} = -\Delta U \quad (\text{work done by a conservative force})$$

$U$  - potential energy (J)

$$\Delta K + \Delta U = 0 \quad (\text{cons. of energy})$$

$\Delta K$  - kinetic energy (J)

$$U_{1,2} = \frac{q_1 q_2}{4\pi \epsilon_0 r} \quad (\text{potential energy of two charges})$$

$r$  - distance between charges (J)

$$U_0 = \sum_{i=1}^N U_{i0} = \sum_{i=1}^N \frac{q_i q_0}{4\pi \epsilon_0 r_{i0}} \quad (\text{multiple charge potential})$$

$$V(\vec{r}) = \lim_{q_0 \rightarrow 0} \left( \frac{U_{i0}}{q_0} \right) \quad (\text{electric potential})$$

$V(\vec{r})$  - electric potential (V)

$q_0$  - test charge

$$V(\vec{r}) = \frac{q}{4\pi \epsilon_0 |\vec{r} - \vec{r}'|} \quad (\text{potential of a point charge})$$

$r'$  - position of point charge (m)

$$V_{ab} = V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} \quad (\text{electric potential})$$

line integral here.

$$\vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} = -\vec{\nabla} V \quad (\text{potential fields})$$

$\vec{\nabla}$  - gradient operator.

$$C = \frac{Q}{V} \quad (\text{capacitor})$$

$C$  - capacitance (F)

$Q$  - charge (coulombs)

$V$  - voltage / potential difference (V)

$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel plate capacitor})$$

$d$  - distance between plates (m)

$A$  - surface area of plates ( $m^2$ )

$$C = \frac{Q}{V_{ab}} = \frac{2\pi \epsilon_0 L}{\ln\left(\frac{R_b}{R_a}\right)} \quad (\text{cylindrical capacitor})$$

$R_b$  - radius from centre to outside plate (m)

$R_a$  - radius from centre to inside plate (m)

$L$  - length of capacitor (m)

$$C = \frac{Q}{V_{ab}} = \frac{\epsilon_0 4\pi r_a r_b}{r_b - r_a} \quad (\text{spherical capacitor})$$

$r_a$  - radius of 1<sup>st</sup> plate (m)

$r_b$  - radius of 2<sup>nd</sup> plate (m)

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (\text{capacitors in series})$$

$$C_{eq.} = C_1 + C_2 \quad (\text{capacitors in parallel})$$

$C_{eq.}$  - equivalent capacitance (F)

$$u = \sum_{i=1}^n \frac{1}{2} q_i V_i \quad (\text{total potential energy of } n \text{ charges})$$

$$u = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} \quad (\text{energy stored in capacitor})$$

$$u = \frac{u'}{V} = \frac{1}{2} \epsilon_0 E^2 \quad (\text{energy density for electric field})$$

$u$  - energy density ( $Jm^{-3}$ )

$u'$  - energy (J)

$V$  - volume ( $m^3$ )

$E$  - magnitude of electric field ( $Nc^{-1}$ )

$$\epsilon_r = \frac{C}{C_0} = \frac{\epsilon}{\epsilon_0} \quad (\text{dielectric constant})$$

$C_0$  - capacitance separated by vacuum (F)

$C$  - capacitance separated by dielectric (F)

$\epsilon$  - permittivity of material



$$\vec{P}(\vec{r}) = \frac{1}{V} \sum_i \vec{p}_i \quad (\text{polarization density})$$

$\vec{P}(\vec{r})$  - polarization density ( $\text{Cm}^{-2}$ )

$V$  - volume ( $\text{m}^3$ )

$\vec{p}_i$  - electric dipole moment ( $\text{Cm}$ )

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (\text{electric displacement vector})$$

$\vec{D}$  - displacement vector ( $\text{Cm}^{-2}$ )

$\vec{E}$  - electric field strength ( $\text{NC}^{-1}$ )

$$\vec{P} = (\epsilon_r - 1) \epsilon_0 \vec{E} = \chi_e \epsilon_0 \vec{E} \quad (\text{polarization density round 2.})$$

$$\chi_e = \epsilon_r - 1 \quad (\text{electric susceptibility})$$

$$I = \frac{dQ}{dt} \quad (\text{current})$$

$I$  - current (A)

$Q$  - charge (C)

$t$  - time (s)

$$\vec{J} = \frac{I}{A} \hat{z} = nq \vec{v}_d \quad (\text{current density vector})$$

$\vec{J}$  - ( $\text{Am}^{-2}$ )

$A$  - cross-sectional area ( $\text{m}^2$ )

$n$  - number of free charges

$\vec{v}_d$  - drift velocity ( $\text{m}^{-1}$ )

$$\vec{E} = \rho \vec{J} \quad (\text{Ohm's law})$$

$$\rho = \frac{E}{J} \quad (\text{resistivity relation})$$

$\rho$  - resistivity ( $\Omega\text{m}$ )

$$\sigma = \frac{1}{\rho} \quad (\text{conductivity relation})$$

$\sigma$  - conductivity ( $\Omega^{-1}\text{m}^{-1}$ )

$$R = \frac{\rho L}{A} \quad (\text{resistance for conductor of length } L)$$

$A$  - cross-sectional area ( $\text{m}^2$ )

•  $V = IR$  (Ohm's law)

$V$  - potential difference (V)

$R$  - resistance ( $\Omega$ )

$I$  - current (A)

•  $R_{eq} = R_1 + R_2$  (resistors in series)

•  $R_{eq}^{-1} = \frac{1}{R_1} + \frac{1}{R_2}$  (resistors in parallel)

$R_{eq}$  - equivalent resistance ( $\Omega$ )

•  $P = V_{ab} I$  (power dissipated)

$V_{ab}$  - potential difference across circuit element ab.

$P$  - power (W)

•  $P = I^2 R = \frac{V_{ab}^2}{R}$  (power across resistor)

•  $\frac{P}{V} = \rho J^2$  (power per unit volume)

$V$  - volume ( $m^3$ )

$\rho$  - resistivity ( $\Omega m$ )

$J$  - current density ( $A m^{-2}$ )

•  $V = \mathcal{E} - rI$  (internal resistance and emf.)

$\mathcal{E}$  - electromotive force (V)

$r$  - internal resistance of power supply ( $\Omega$ )

•  $\sum_{i, in} I_i = \sum_{i, out} I_i$  (Kirchhoff's Junction rule)

$I_{i(in)}$  - current into a junction (A)

$I_{i(out)}$  - current out of a junction (A)

•  $\sum_i \mathcal{E}_i = \sum_i V_j$  (Kirchhoff's loop rule)

$\mathcal{E}_i$  - sum of emfs (V)

$V_j$  - sum of voltage drops (V)



Capacitor charging equations:

$$I = I_0 e^{-\frac{t}{RC}}$$

$$Q = Q_0 (1 - e^{-\frac{t}{RC}})$$

$$V = V_0 (1 - e^{-\frac{t}{RC}})$$

Capacitor discharging equations:

$$I = I_0 e^{-\frac{t}{RC}}$$

$$V = V_0 e^{-\frac{t}{RC}}$$

$$Q = Q_0 e^{-\frac{t}{RC}}$$

$I_0$  - initial current (A)

$V_0$  - final voltage (V)

$Q_0$  - final charge (C)

$t$  - time (s)

$R$  - resistance ( $\Omega$ )

$C$  - capacitance (F)

$$\vec{F} = q \vec{v} \times \vec{B} \quad (\text{Lorentz force})$$

$q$  - charge (C)

$\vec{v}$  - velocity ( $\text{ms}^{-1}$ )

$\vec{B}$  - magnetic field (T)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (\text{with electric field contribution})$$

$\vec{E}$  electric field ( $\text{Nc}^{-1}$ )

$$\Phi_B = \iint_S \vec{B} \cdot d\vec{s} \quad (\text{magnetic flux})$$

$\Phi_B$  - magnetic flux (wb)

$d\vec{s}$  - surface element ( $\text{m}^2$ )

Surface integral here.

- $\oint_S \vec{B} \cdot d\vec{s} = 0$  (solenoidal condition)

- $R_L = \frac{mv}{|q|B}$  (Larmor radius)

$R_L$  - Larmor radius (m)

$m$  - mass of charge (kg)

$v$  - perpendicular velocity ( $\text{ms}^{-1}$ )

- $d\vec{F} = I d\vec{l} \times \vec{B}$  (force on section  $d\vec{l}$  of wire)

$I$  - current (A)

- $\vec{F} = BIL \hat{n}$  (force on moveable crossbar in  $B$ )

$L$  - length of crossbar.

- $\vec{\tau} = \vec{\mu} \times \vec{B}$  (torque on rectangular current loop)

$\vec{\mu}$  - magnetic dipole moment ( $\text{Cm}^2\text{s}^{-1}$ )

$\vec{\tau}$  - torque (Nm)

- $\vec{\mu} = NIA \hat{n}$  (magnetic dipole moment)

$N$  - number of turns

$I$  - current (A)

$A$  - area of loop ( $\text{m}^2$ )

$\hat{n}$  - direction perpendicular to plane of loop

- $\vec{B}(\vec{r}, t) = \frac{\mu_0}{4\pi r^3} \vec{v}(t) \times (\vec{r} - \vec{r}'(t))$  (Biot-Savart law)

$\mu_0$  - permeability of free space

$\vec{v}(t)$  - velocity of charge ( $\text{ms}^{-1}$ )

$\vec{r}(t)$  - position of field strength (m)

$\vec{r}'(t)$  - position of charge (m)

- $\frac{|\vec{F}_B|}{|\vec{F}_E|} = \left(\frac{v}{c}\right)^2$  (ratio of forces felt by two moving charges)

$\vec{F}_B$  - magnetic force (N)

$\vec{F}_E$  - electric force (N)

$c$  - speed of light ( $\text{ms}^{-1}$ )



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int I \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad (\text{Biot-Savart law for a wire})$$

$\vec{r}'$  - position vector of wire (m)

$\vec{r}$  - position vector of strength of B (m)

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi R} \hat{\phi} \quad (\text{magnetic field from straight wire})$$

R - radial distance from centre of wire (m)

$\hat{\phi}$  - direction given by right-hand rule

$$\vec{B}(\vec{r}) \approx \frac{\mu_0}{4\pi r^3} [3(\vec{\mu} \cdot \hat{r}) \hat{r} - \vec{\mu}] \quad (\text{Ideal magnetic dipole field})$$

$\vec{\mu}$  - magnetic dipole moment ( $\text{Cm}^2 \text{s}^{-1}$ )

$\hat{r}$  - direction of position vector (m)

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \quad (\text{Ampère's law})$$

line integral around a closed loop

$I_{\text{enc}}$  - enclosed current by Amperian loop (A)

$$\vec{B}_{\text{outside}} = 0; \quad \vec{B}_{\text{inside}} = \mu_0 n I \hat{z} \quad (\text{solenoidal magnetic field})$$

n - number of turns

I - current (A)

$$\vec{B} = \mu_r \vec{B}_0 \quad (\text{magnetic field inside materials})$$

$\vec{B}_0$  - external magnetic field (T)

$\mu_r$  - relative permeability

$$\mu_r < 1 \quad (\text{diamagnetic material})$$

$$\mu_r > 1 \quad (\text{paramagnetic material})$$

$$\mu_r \gg 1 \quad (\text{ferromagnetic material})$$

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} \quad (\text{Faraday's law})$$

N - number of turns

t - time (s)

$\Phi_B$  - magnetic flux (Wb)

$\mathcal{E}$  - emf (V)

- $\mathcal{E} = \oint_{\text{loop}} (\vec{v} \times \vec{B}) \cdot d\vec{l}$  (motional emf in closed loop)

- $\mathcal{E} = - \frac{d\Phi_B}{dt} = \oint_c \vec{E} \cdot d\vec{l}$  (stationary emf in closed loop)

- $u = \frac{u'}{v} = \frac{B^2}{2\mu_0}$  (energy density of solenoid)

$u$  - energy density ( $\text{J m}^{-3}$ )

$v$  - volume ( $\text{m}^3$ )

$u'$  - potential energy (J)

- $I_D = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{S}$  (displacement current)

$I_D$  - displacement current (A)

- $L = \frac{N\Phi_B}{I}$  (self-inductance)

$L$  - self-inductance (H)

- $\mathcal{E} = -L \frac{dI}{dt}$  (emf in an inductor)

- $u = \frac{1}{2} LI^2$  (energy stored by inductor)

- $L_{eq} = L_1 + L_2$  (inductors in series)

- $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$  (inductors in parallel)

$L_{eq}$  - equivalent inductance (H)

- $\frac{N_s}{N_p} = \frac{V_s}{V_p}$  (Transformer equation)

- $\mathcal{E} = \frac{I_s V_s}{I_p V_p}$  (efficiency of transformer)

$N_p$  - no. of turns of primary coil

$N_s$  - no. of turns of secondary coil

$V_p$  - voltage across primary coil (V)

$V_s$  - voltage across secondary coil (V)

$\mathcal{E}$  - efficiency



$$V(t) = V_0 \cos \omega t = \operatorname{Re} (V_0 e^{i\omega t}) \quad (\text{Alternating voltage})$$

$\omega$  - angular velocity ( $s^{-1}$  or  $\text{rad s}^{-1}$ )

$t$  - time (s)

$V_0$  - peak voltage (V)

$$\tilde{V} = \tilde{Z} \tilde{I} \quad (\text{complex impedances})$$

$\tilde{V}$  - complex voltage (V)

$\tilde{I}$  - complex current (I) } phasors.

$\tilde{Z}$  - complex impedance ( $\Omega$ )

$$\tilde{Y} = \frac{1}{\tilde{Z}} \quad (\text{complex admittance})$$

$\tilde{Y}$  - complex admittance (S)

$$\tilde{Z}_R = R \quad (\text{complex impedance of resistor})$$

$$\tilde{Z}_C = \frac{1}{j\omega C} \quad (\text{complex impedance of capacitor})$$

$j$  is the same as  $i$ , i.e.  $j = \sqrt{-1}$

$$\tilde{Z}_L = j\omega L \quad (\text{complex impedance of inductor})$$

$$\tilde{Z}_{eq} = \tilde{Z}_1 + \tilde{Z}_2 \quad (\text{complex impedances in series})$$

$$\frac{1}{\tilde{Z}_{eq}} = \frac{1}{\tilde{Z}_1} + \frac{1}{\tilde{Z}_2} \quad (\text{complex impedances in parallel})$$

$\tilde{Z}_{eq}$  = equivalent complex impedance ( $\Omega$ )

$$I_{rms} = \frac{I_0}{\sqrt{2}} \quad (\text{root mean square current})$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} \quad (\text{root mean square voltage})$$

$$P_{rms} = V_{rms} I_{rms} \quad (\text{mean power})$$

$$P = \frac{1}{2} V_0 I_0 \cos \phi = V_{rms} I_{rms} \cos \phi \quad (\text{mean power})$$

$\cos \phi$  - power factor.

$$\cos \phi = \frac{\operatorname{Im}(\tilde{Z})}{\operatorname{Re}(\tilde{Z})} \quad (\text{power factor})$$

$$E = BAN \omega \sin \omega t \quad (\text{emf in rotating coil})$$

$N$  - no. of turns

$\omega$  - angular velocity ( $s^{-1}$ )

## Quantum Phenomena

- $W_{\text{total}} = -eV_0 = E_{k \text{ max}} = \frac{1}{2}mv^2$  (Work done by electron)
  - $v$  - maximum velocity of electron ( $\text{ms}^{-1}$ )
  - $m$  - mass of electron ( $\text{kg}$ )
  - $V_0$  - stopping potential ( $\text{V}$ )
  - $e$  - charge on electron ( $\text{C}$ )
- $E = hf = hc/\lambda$  (Photoelectric eqn)
  - $E$  - energy of photon ( $\text{eV}$  or  $\text{J}$ )
  - $h$  - Planck's constant ( $\text{Js}$ )
  - $f$  - frequency of photon ( $\text{Hz}$ )
  - $c$  - speed of light ( $\text{ms}^{-1}$ )
  - $\lambda$  - wavelength of photon ( $\text{m}$ )
- $hf > \phi$  (threshold frequency)
  - $f$  - frequency required to emit photon ( $\text{Hz}$ )
  - $\phi$  - work function ( $\text{eV}$  or  $\text{J}$ )
- $E = pc$  (photon momentum)
  - $p$  - momentum ( $\text{kgms}^{-1}$ )
- $eV_{\text{ac}} = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$  (Bremsstrahlung equation)
  - $V_{\text{ac}}$  - accelerating voltage ( $\text{V}$ )
  - $f_{\text{max}}$  - maximum frequency ( $\text{Hz}$ )
  - $\lambda_{\text{min}}$  - minimum wavelength ( $\text{m}$ )
- $\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)$  (Compton scattering)
  - $\lambda'$  - wavelength of scattered radiation ( $\text{m}$ )
  - $\lambda$  - wavelength of incident radiation ( $\text{m}$ )
  - $m$  - electron rest mass ( $\text{kg}$ )
  - $\phi$  - angle of deflection ( $\text{rad}$  or  $^\circ$ )
- $E_{\text{min}} = 2mc^2$  (pair production for electron)
  - $E_{\text{min}}$  - minimum energy for pair production ( $\text{eV}$  or  $\text{J}$ )
- $\hbar = \frac{h}{2\pi}$  (reduced Planck constant)
  - $h$  - Standard Planck constant ( $\text{Js}$ )



$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (\text{Heisenberg uncertainty principle, pos + momentum})$$

$\Delta x$  - position uncertainty (m)

$\Delta p$  - momentum uncertainty ( $\text{kgms}^{-1}$ )

$\hbar$  - reduced Planck constant (Js)

$$\Delta t \Delta E \geq \frac{\hbar}{2} \quad (\text{Heisenberg uncertainty principle, energy + time})$$

$\Delta t$  - time uncertainty (s)

$\Delta E$  - energy uncertainty (J)

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad (\text{De Broglie wavelength})$$

$p$  - momentum of particle ( $\text{kgms}^{-1}$ )

$\lambda$  - de Broglie wavelength (m)

$$\lambda = \frac{h}{\sqrt{2meV_{ab}}} \quad (\text{de Broglie wavelength for electron})$$

$V_{ab}$  - accelerating voltage (V)

$e$  - charge on electron (C)

$m$  - mass of electron (kg)

$$\Delta E = E_i - E_f = hf \quad (\text{energy levels})$$

$\Delta E$  - change in energy (J)

$E_i$  - initial level energy (J)

$E_f$  - final level energy (J)

$$L_n = mv_n r_n = n \frac{h}{2\pi} \quad (\text{Quantization of angular momentum})$$

$L_n$  - orbital angular momentum at level  $n$  ( $\text{kgm}^2\text{s}^{-1}$ )

$m$  - electron mass (kg)

$v_n$  - electron speed (ms)

$r_n$  - radius of orbit (m)

$n$  - principal quantum number ( $n \in \mathbb{N}$ )

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \quad (\text{radius of } n^{\text{th}} \text{ orbit in Bohr model.})$$

$\epsilon_0$  - permittivity of free space ( $\text{Fm}^{-1}$ )

$$v_n = \frac{1}{\epsilon_0} \frac{e^2}{2nh} \quad (\text{orbital velocity of } n^{\text{th}} \text{ orbit in Bohr model.})$$

- $r_n = n^2 a_0$  (Bohr radius)

$a_0$  - Bohr radius (m)

- $E_n = -\frac{hcR}{n^2}$  (total energy)

$R$  - Rydberg constant ( $m^{-1}$ )

$c$  - speed of light ( $ms^{-1}$ )

- $\lambda_m T = 2.90 \times 10^{-3} mK$  (Wien displacement law for blackbodies.)

$\lambda_m$  - peak wavelength (m)

$T$  - temperature of blackbody (K)

- $I(\lambda) = \frac{2\pi^5 c k_B^4 T^5}{15 h^3 \lambda^5}$  (Rayleigh-Jeans law)

$k_B$  - Boltzmann constant ( $JK^{-1}$ )

$I(\lambda)$  - intensity as a function of wavelength ( $Wm^{-2}$ )

- $I(\lambda) = \frac{2\pi^5 h c^2}{15 \lambda^5} \cdot \frac{1}{(e^{\frac{hc}{\lambda k_B T}} - 1)}$  (Planck radiation law.)

$I(\lambda)$  - spectral emittance ( $Wm^{-3}$ )

$T$  - absolute temperature of black body (K)

- $E = \hbar\omega$  (energy / angular velocity relation)

$E$  - energy of particle (J/eV)

$\omega$  - angular velocity ( $rad s^{-1}$ )

- $p = \hbar k$  (momentum / wavenumber relation)

$p$  - momentum ( $kgms^{-1}$ )

$k$  - wavenumber

- $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + u(x) \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$  (General Schrödinger eqn.)

$m$  - mass of particle (kg)

$\hbar$  - reduced planck constant (Js)

$\Psi(x,t)$  - wavefunction

$u(x)$  - potential energy function (J)

$i$  -  $\sqrt{-1}$



$$\Psi(x, t) = A [\cos(kx - \omega t) + i \sin(kx - \omega t)] = A e^{i(kx - \omega t)} \quad (\text{Wavefunction})$$

$A$  - amplitude (can be complex)

$$\Psi(x, t) = \Psi(x) e^{-\frac{iEt}{\hbar}} \quad (\text{time-dependent wavefunction})$$

$t$  - time (s)

$E$  - energy (J)

$$-\frac{\hbar^2}{2m} \frac{d^2 \Psi(x)}{dx^2} + u(x) \Psi(x) = E \Psi(x) \quad (\text{time-independent Schrödinger eqn})$$

$\Psi(x)$  - time-independent wavefunction

$m$  - mass (kg)

$$E_n = \frac{p_n^2}{2m} = \frac{n^2 \hbar^2}{8mL^2} = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (\text{energy levels for a particle in a box.})$$

$p_n$  - momentum ( $\text{kgms}^{-1}$ )

$m$  - mass of particle (kg)

$L$  - width of box (m)

$n \in 1, 2, 3, \dots$

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1 \quad (\text{Normalization condition})$$

$\Psi(x)$  - time-independent wavefunction

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad (\text{Particle in box time-independent wavefunction})$$

$L$  - length of box (m)

$n \in 1, 2, 3, \dots$