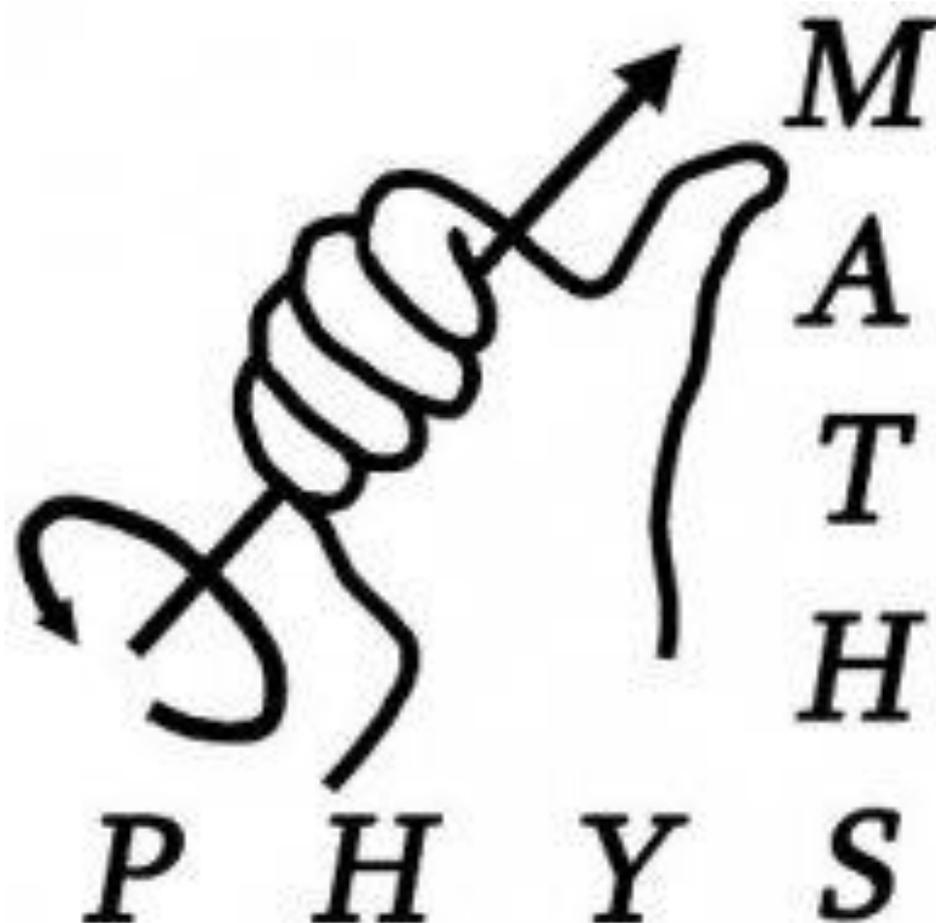


Px121: Thermal physics 1



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September 2011*

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Disclaimer:

This guide has been constructed to aid revision for the module in question. All content has been checked I but cannot guarantee it is all accurate. Hopefully you will find it useful and a good place to start or to compliment your revision. Remember, using your own notes and other sources will help you to revise and will reduce the likelihood of copying any mistakes in this guide. Good luck! 😊

Introduction

Imagine 1 mole of gas, sat in some volume. At this instant there would be about 6×10^{23} particles in this volume. If we want to fully describe this system we would have to take account of all the information about each particle, which themselves, are under time dependant motion. This causes a problem, how can we possibly describe this quite frankly, simple system when there are so many variables to account for? This is where classical thermodynamics comes in, and is the basis of the course thermal physics 1.

Systems, processes and lots of definitions

A **system** is basically a small bit of the universe which we are trying to describe using thermodynamics. Such a system could be, a car engine, a refrigerator or even the weather (shame the weather forecasts can't model this system accurately!)

At some given time, the physical condition of the system is called its **state** and the equations we use to describe the system and its state are called, the **equations of state**.

In classical thermodynamics, Temperature (T), pressure (p), volume (V) and mass (m) are called **State variables**,

Now, all systems can be distinguished into two types, **open and closed systems**.

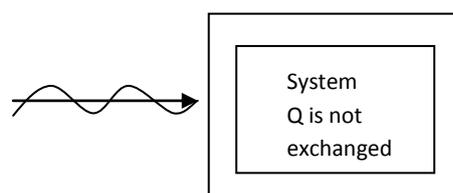
Open systems can exchange **energy AND particles** (mass) with its surroundings.

Closed systems can **ONLY exchange energy**.

Until thermal physics 2 next year, this module will only consider closed systems (this is a good thing).

For a system to be closed, we must separate it, we do this using walls. These walls can either be, **adiabatic or diathermal**.

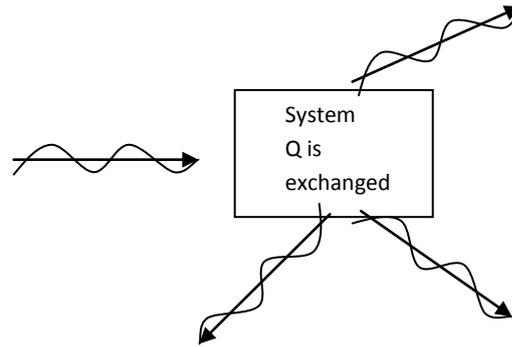
Adiabatic walls prevent the transfer of heat energy (Q) from the system, to the environment.



A vacuum is an example of this.

Diathermal walls allow the transfer of heat energy (Q) from the system, to the environment.

Metals are good examples of this.



Now for processes. A process is simply something which happens to a system. There are specific processes which can happen to a system under certain conditions, these are listed below and are very useful to know.

Isothermal: a process which occurs at constant temperature

Isochoric: a process which occurs at constant volume

Isobaric: a process which occurs at constant pressure

Adiabatic: a process which occurs with no heat exchange ($dQ = 0$)

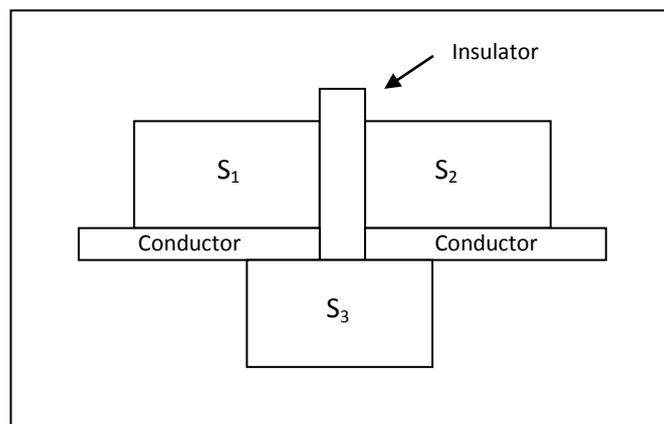
Isentropic: a process which occurs with constant entropy (less important, just for completeness)

You will also need to know what thermal equilibrium is defined as.

Thermal equilibrium: two systems are in thermal equilibrium if their state variables do not change when in contact with each other via a diathermal wall.

Zeroeth Law of Thermodynamics

Using the figure below... "If systems S_1 and S_2 are separated in thermal equilibrium with system S_3 then systems S_1 and S_2 are **also** in thermal equilibrium."



This is the Zeroeth law of thermodynamics!

Temperature

Temperature is a variable of state and is a physical property of a system which is related to the kinetic energy of the atoms in the system.

Another concept to understand is the internal energy (U) of a system. Internal energy is the sum of all the energies in a system (translational, rotational and vibrational energies)

Temperature is **only** dependant on the internal energy! Hence if we increase internal energy of a system, by adding heat (Q), we increase the system's temperature.

Thermometry

How do we measure temperatures?

For some system, say there is a quantity we can measure, call it X. This quantity varies with temperature T.

Then we would like to work out the following;

$$X(T) = X_0 + cT \quad (1)$$

So if we know, X_0 and c , which are just constants we can calibrate a curve for the system.

To do this, we need to fixed points which are easy to reproduce.

The Celsius temperature scale uses two points

- Ice point (0°C)
- Steam point (100°C)

So then we can measure the quantity X at each point. Hence;

$$T = \frac{100(X - X_{ice})}{X_{steam} - X_{ice}} \text{ } ^\circ\text{C} \quad (2)$$

Examples of quantities of X include:

- Expansion of solids or liquids

$$\Delta V = V - V_0 = \beta V_0 \Delta T \quad (3)$$

Where, β , is the volume coefficient of expansion, T is temperature and V_0 is some zero point volume.

- Change in electrical resistance

$$\Delta R = \gamma R_0 \Delta T \quad (4)$$

Where, γ , is some coefficient of this variable, R_0 is some zero point resistance, and T is temperature.

- Thermocouple, such a device is used in some kettles. Different metals form a reference junction.

We can also measure the temperature of an object from the radiation it emits. This is from the Wien displacement law:

$$\lambda_m T = 2.9 \times 10^{-3} \text{ mK} \quad (5)$$

NOTE: the units are metres Kelvin **NOT** milli-Kelvin

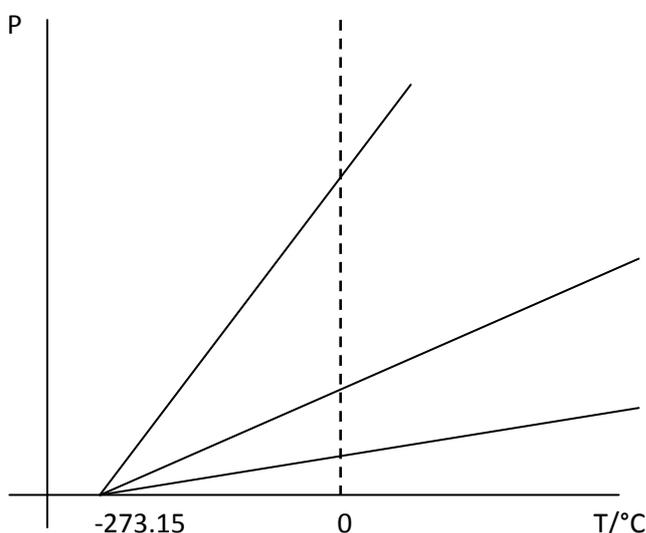
Absolute zero

We can never actually achieve absolute zero, this is because particles would violate the Heisenberg's uncertainty principle. However, absolute defines the zero point on the Kelvin scale.

$$\text{Absolute zero} = -273.15^\circ\text{C} = 0\text{K} \quad (6)$$

This is found from extrapolating lines from a pressure, temperature graph for different values of, n. (see figure below). We have used the ideal gas equation:

$$PV = nRT \quad (7)$$



We can rearrange equation (7) to give us the equations of the lines, equation (8).

$$\frac{P}{T} = \frac{nR}{V} \quad (8)$$

This gives the linear lines and hence for different values of n, we obtain different gradients, but they all extrapolate to absolute zero!

You may also notice, for the Kelvin scale, we only need **1 reference point!**

Triple point of water

We can use the triple point of water as a reference point. This triple point occurs at 0.01°C and 610Pa and is the point where, solid, liquid and vapour are all in equilibrium. This occurs at the triple point temperature:

$$T_{tr} = 273.16 \text{ K} \quad (9)$$

When gases are behaving ideally we find that:

$$\frac{P}{T} = \text{constant} \quad (10)$$

Helium exhibits this behaviour at low pressures.

Thermal expansion

Linear thermal expansion

- Metals tend to have coefficients of expansion of around 10^{-5} K^{-1}
- Plastics tend to have coefficients of expansion of around 10^{-4} K^{-1} or more.

For linear expansions, for example, length, the expansion, α is given by:

$$\alpha \approx \frac{1}{L_0} \frac{\Delta L}{\Delta T} \quad (11)$$

Where, L is the linear dimension in question, and L_0 is simply the initial length etc.

- **Isotropic** materials experience the same expansion in all directions.

Equation (11) works fine for limited temperature ranges, but a better description would be to allow, L , to vary as a function of temperature, hence $L(T)$. It follows:

$$\alpha = \alpha(T) = \frac{1}{L(T)} \frac{dL}{dT} \quad (12)$$

$$\alpha dT = \frac{dL}{L(T)} \quad (13)$$

$$\alpha (T - T_0) = \ln\left(\frac{L}{L_0}\right) \quad (14)$$

We arrive at equation (14) by integrating equation (13).

Volume thermal expansion

Like for linear expansion, we arrive at a volume coefficient of expansion, β , as follows:

$$\beta \approx \frac{1}{V_0} \frac{\Delta V}{\Delta T} \quad (15)$$

And if $V = L^3$ and the material is isotropic, then from

$$dV = 3L^2 dL \quad (16)$$

We can then show from equations (11) and (15):

$$\beta = 3\alpha \quad (17)$$

The reason for expansion is because the mean atomic position increases as the temperature increases and hence the mean inter-atomic spacing increases...therefore expansion.

Heat capacity

Heat flow (Q) into a substance increases its internal energy and hence, its temperature. The only exception to this is if the system did an equivalent amount of work as to that of the heat input.

For a substance of heat capacity, C:

$$\Delta T = \frac{Q}{nC} \quad (18)$$

Where n, is the number of moles of substance. In this case, C is the molar heat capacity.

Equally, instead of moles, n, we can use mass, m:

$$\Delta T = \frac{Q}{mC_m} \quad (19)$$

Where, m, is the mass of the substance. In this case, C_m is the thermal capacity.

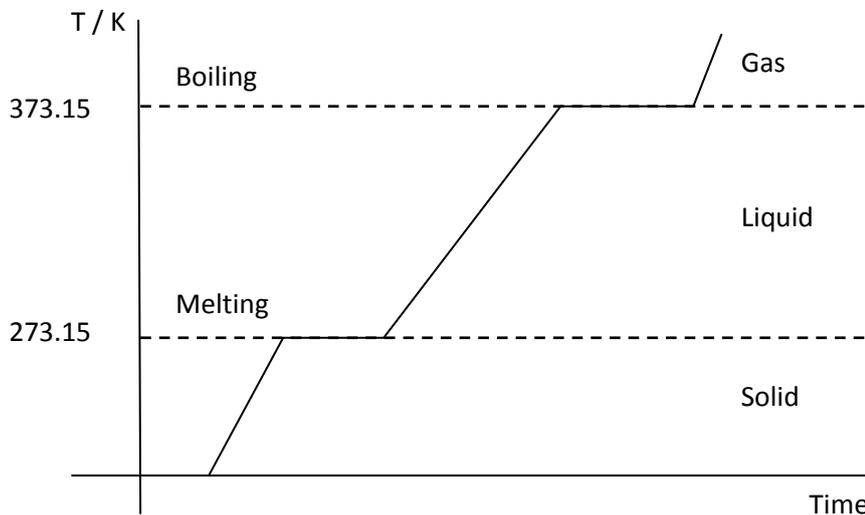
Phase changes

In a phase change, heat input into a substance is used for a change in the atomic arrangement and the binding on the atoms in the substance.

Latent heat, is the amount of heat required to cause a phase change (per mole, or per kg)

- Latent heat of fusion, L_f , e.g. Melting a solid.
- Latent heat of vaporization, L_v , e.g. Converting a liquid to a gas.

The figure below shows an example of these phase changes (very useful to recognise the shape).



This is an example of the phase changes for water which initially was ice which was then heated.

Heat transfer

Conduction

This occurs in solids or circulation free fluids. We consider the rate of flow of heat, which is usually called the **heat flux**, given as follows;

$$\frac{dQ}{dt} = -\kappa A \frac{dT}{dx} \quad (20)$$

Where, Q is the heat, κ is the thermal conductivity, and T is the temperature. The smaller the value of thermal conductivity, the better the thermal insulator it is. For example, polystyrene's thermal conductivity is around 0.01 where as silver is around 430. This is expected as **we all know** metals are good conductors of heat...right?

Convection

This occurs in fluids where circulation can occur, heat flow occurs because:

- Local changes in density
- Local changes in temperature, T

Heat transfer from a heater for example is given by:

$$\frac{dQ}{dt} = hA\Delta T \quad (21)$$

Here, h is the convection coefficient which depends on the material's properties and the shape and size of the system. A, is the area of the heater.

Quite typically,

$$h \text{ is proportional to } \Delta T^{\frac{1}{4}} \quad (22)$$

Radiation

A surface at a temperature, T, will radiate a power, P, which is proportional to the area, A and T^4 .

$$P = \frac{dQ}{dt} = e\sigma AT^4 \quad (23)$$

Here, e is the surface emissivity (**e = 1 for a black body**) and σ , is the Stefan-Boltzmann constant = $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$.

Kinetic theory of gases

Most important...ideal gas equations:

$$pV = nRT \quad \text{and} \quad pV = NK_B T \quad (24 \text{ and } 25)$$

We use (24) when dealing with moles of a gas, and (25) when we are dealing with a number of particles. R = the gas constant = 8.31 Jmole^{-1} and K_B = Boltzmann's constant = $1.38 \times 10^{-23} \text{ JK}^{-1}$.

Assumptions:

- Neglects finite molecular size
- Neglects interactions between molecules (i.e. viscous effects)

You may notice from 24 and 25, that, (where N_A is Avogadro's constant):

$$k_B = \frac{R}{N_A} \quad (26)$$

To account for the two assumptions above we modify the equations, this is the Van der Waal's equation (27):

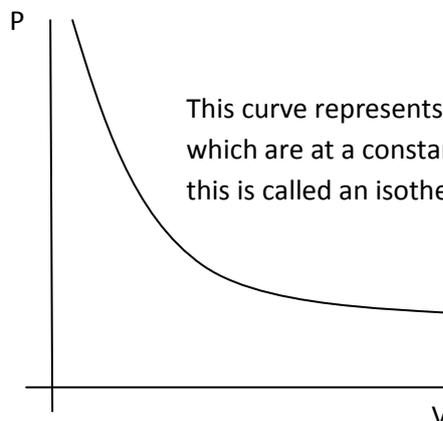
$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT \quad (27)$$

- $V-nb$ accounts for the finite volumes (i.e. total volume minus n times molecules' volume)
- an^2/V^2 accounts for intermolecular forces.

a and b are gas dependent constants.

p-V diagrams

We like to plot $p - V$ diagrams as we can easily show hyperbolic curves along the diagram which represent isotherms i.e. constant temperature lines (like equal potential lines in gravity fields)



This curve represents all the points which are at a constant temperature this is called an isotherm.

We will come back to these with thermodynamic models.

Kinetic-molecular model of an ideal gas

The total momentum change for a particle which hits a wall and comes back is simply:

$$\Delta p = 2m\bar{v}_x \quad (28)$$

Where, m is the mass of the particle and \bar{v}_x is the average x component of velocity.

For some volume of space where the gas is contained, V , in which it contains N particles we find the number of collisions, n_{col} is given by:

$$n_{col} = \frac{1}{2} \frac{N}{V} A \bar{v}_x dt \quad (29)$$

This is basically saying, the number of collisions is the amount of volume per particle (N/V) times the area cross section, A , times the distance travelled in a time dt which is \bar{V}_x .

Equation (29) lead to the total change of momentum, dq as follows:

$$dq = n_{col} 2m\bar{V}_x = \frac{N}{V} Am\bar{V}_x^2 dt \quad (30)$$

But! Force = rate of change of momentum (Newton's second law)

$$F = \frac{dq}{dt} = \frac{N}{V} Am\bar{V}_x^2 \quad (31)$$

We also know the pressure = force per unit area therefore:

$$p = \frac{F}{A} = \frac{N}{V} m\bar{V}_x^2 \quad (32)$$

Now taking this a bit further, we know that:

$$\bar{V}^2 = \bar{V}_x^2 + \bar{V}_y^2 + \bar{V}_z^2 \quad (33)$$

Where the root mean squared speed (rms speed) is given by

$$\bar{V} = \sqrt{\bar{V}^2} \quad (34)$$

But if $\bar{V}_x = \bar{V}_y = \bar{V}_z$, then:

$$\bar{V}_x^2 = \frac{1}{3}\bar{V}^2 \quad (35)$$

We can re-write (32) using (35):

$$pV = nRT = \frac{1}{3}Nm\bar{V}^2 = \frac{2}{3}K_{tr} \quad (36)$$

Where K_{tr} is the average translation energy of the gas.

From equation (24 and 25), equation (36) directly leads to an expression from the **average translational kinetic energy** of a gas molecule at a temperature, T .

$$E_{tr} = \frac{3}{2}K_B T \quad (37)$$

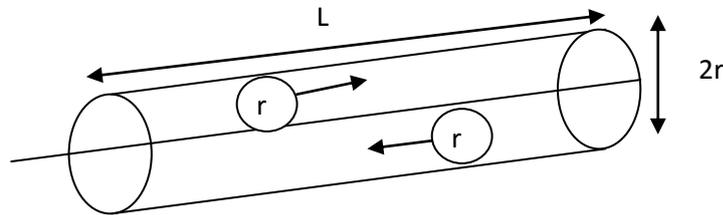
With a root mean squared speed (rms) of:

$$V_{rms} = \sqrt{\frac{3K_B T}{m}} \quad (38)$$

Mean free path

Gas molecules are always colliding with each other and/or the walls of their container. The average distance between collisions is called the **mean free path, denoted by λ** .

However to work this out, we need to know the average time between collisions. Consider the following:



The molecules have a radius of, r . Therefore a collision occurs if they approach within, $2r$, of each other. Now, if each molecule has a rms of equation (34) then their relative rms speeds will be:

$$V'_{rms} = \sqrt{2}V_{rms} \quad (39)$$

Now, we can rewrite the length of our cylinder, L , as (as distance = speed x time):

$$L = \sqrt{2}V_{rms}dt \quad (40)$$

Supposing there are dN collisions in time dt we arrive:

$$dN = 4\pi r^2 \sqrt{2}V_{rms}dt \frac{N}{V} \quad (41)$$

This is simply, Volume of the cylinder ($4\pi r^2 \sqrt{2}V_{rms}dt$, i.e. $4\pi r^2 L$) time the number of molecules per unit volume (N/V).

Hence!, finally!, we arrive at a collision rate by dividing by, dt :

$$\frac{dN}{dt} = 4\pi r^2 \sqrt{2}V_{rms} \frac{N}{V} \quad (42)$$

Phew, now for the easy bit, **mean time between collisions** is just the $1/\text{collision rate}$:

$$t = \frac{1}{\frac{dN}{dt}} = \frac{V}{4\pi r^2 \sqrt{2}V_{rms}N} \quad (43)$$

Then the mean free path is just:

$$\lambda = V_{rms}t = \frac{V}{4\pi r^2 N \sqrt{2}} \quad (44)$$

Then from the ideal gas equations (24 and 25):

$$\lambda = \frac{KT}{4\pi r^2 \sqrt{2}p} \quad (45)$$

Notice that:

- λ has NO dependence on speed
- t DOES have dependence on speed

Heat Capacity...again...and you thought you were done!

Sorry but there is plenty more on this, then that's it for this, until next year :P . We define...

- C_v is the heat capacity at constant volume
- C_p is the heat capacity at constant pressure

Now the heat, dQ is given by (constant pressure and n moles of gas)

$$dQ = nC_v dT \quad (46)$$

If all this goes into the K_{tr} equation (36) then we have:

$$dQ = dK_{tr} = \frac{3}{2} nR dT = nC_v dT \quad (47)$$

This directly leads to:

$$C_v = \frac{3}{2} R \quad (48)$$

...For an ideal monatomic gas.

From equipartition of energy (statistical physics) each degree of freedom has an average energy of $0.5K_B T$. But these degrees of freedom are not all excited at the same temperatures.

- Translational mode has 3 thermally excitable degrees of freedom which contributes $3/2 R$ to C_v
- Rotational (as in diatomic molecules) have 2 thermally excitable degrees of freedom which contributes, R to C_v
- Vibrational (as in diatomic molecules) has 1 thermally excitable degree of freedom which contributes, R to C_v

Therefore for a monatomic gas, $C_v = 3/2 R$

For a diatomic gas (with translational and rotational) $C_v = 3/2 R + R = 5/2 R$

For a diatomic gas (with translational and rotational and Vibrational) $C_v = 3/2 R + R + R = 7/2 R$

First law of thermodynamics

$$\Delta U = Q - W \quad (49)$$

REMEMBER THIS!

By convention,

- Heat input, is +Q
- Heat output, is -Q
- Work done BY the system is, +W
- Work done ON the system is, -W

For infinitesimal changes:

$$dU = dQ - dW \quad (50)$$

Thing to remember, U is the **sum** of all internal Kinetic and potential energy, and is a state function.

Reversible processes

- Series of infinitesimally small changes from the original state
- System stays very close to equilibrium
- The work done by a system during reversible process is the maximum possible

Irreversible processes

- Real processes involve finite changes. These cannot be reversed without changing the surroundings
- Large changes cannot be reversed!

Quasi-static processes

- Occurs slowly so the system is considered to be at equilibrium at each moment in time

Work done and more PV diagrams

Work done by gas = force x distance moved by walls. For a reversible volume change dV which leads to a wall moving with area, A , by a distance, dx . The work done therefore is:

$$dW = pAdx = pdV \quad (51)$$

Hence from a volume, V_1 to a volume V_2 :

$$W = \int_{V_1}^{V_2} pdV \quad (52)$$

So, here is the clever bit, you may notice this is the area under the curve on a PV diagram! This is used extensively in this module.

Work done by an isothermal process

Isothermal means, T is constant therefore the work done is:

$$W = \int_{V_1}^{V_2} pdV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right) \quad (53)$$

During this process the heat exchange is equal to the work done:

$$Q = W \quad (54)$$

Work done by an isochoric process

Volume is constant therefore from (52), work done is zero! Nice and simple ☺

$$W = 0 \quad (55)$$

$$Q = -nC_V\Delta T \quad (56)$$

Work done by an isobaric process

Pressure is constant therefore (52) reduces to:

$$W = p\Delta V = P(V_2 - V_1) \quad (57)$$

$$Q = nC_P\Delta T \quad (58)$$

Notice! We have now used C_p NOT C_v .

Work done by an adiabatic process

Adiabatic, therefore no heat transfer hence,

$$Q = 0 \quad (59)$$

$$pV^\gamma = \text{constant} \quad (60)$$

$$TV^{\gamma-1} = \text{constant} \quad (61)$$

$$W = -\Delta U = -\frac{1}{\gamma-1} (p_2V_2 - p_1V_1) \quad (62)$$

Where:

$$\gamma = \frac{C_P}{C_V} \text{ and } C_P - C_V = R \quad (63 \text{ and } 64)$$

Heat capacity, again...again...sorry!

We derive the adiabatic equations above (60 and 61) from considering the definitions of the heat capacities. These are as follows: For constant volume:

$$C_V = \frac{dQ}{dT} \quad (65)$$

If isochoric, $dW = 0$ so $dU = dQ$, hence:

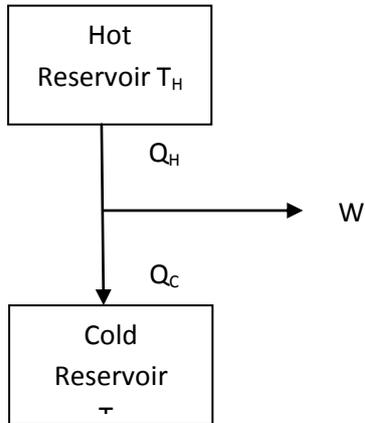
$$C_V = \frac{\partial U}{\partial T} \quad (66)$$

At constant pressure:

$$C_P = \frac{dQ}{dT} \quad (67)$$

The Carnot cycle

A heat engine, converting heat energy into mechanical, is most efficient via a system made up of reversible processes.

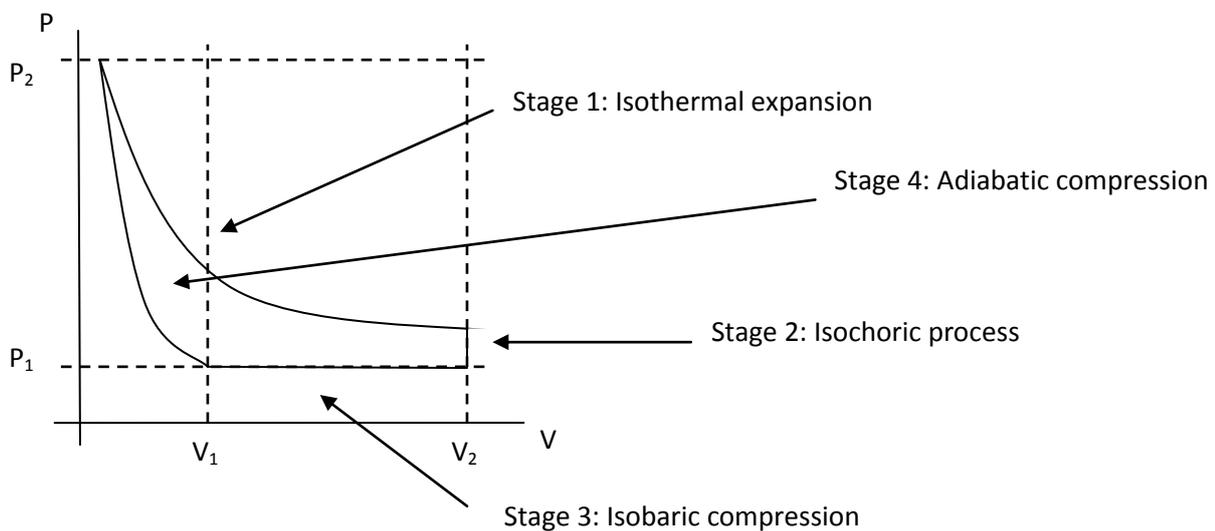


Some heat, Q_H is transferred from the hot reservoir to the engine. Then some heat Q_C is transferred from the engine to the cold reservoir.

The difference in heat is the work done by the engine hence:

$$W = Q_H - Q_C \quad (68)$$

Next I will show an example of a pV diagram for an ideal gas, this is a very typical exam style question!



So we have to now consider and then work out what is going on at each stage!

Stage 1: Isothermal expansion, i.e. p_2, V_0, T_1 expands to, P_1, V_2, T_1 .

$$W = nRT_1 \ln\left(\frac{V_2}{V_0}\right) \quad (69)$$

$$\Delta U = 0 \quad (70)$$

$$Q = W = nRT_1 \ln\left(\frac{V_2}{V_0}\right) \quad (71)$$

Stage 2: Isochoric process, i.e. p_1, V_2, T_1 goes to, p_1, V_1, T_2 .

$$W = 0 \quad (72)$$

$$Q = -nC_V \Delta T \quad (73)$$

$$\Delta U = -nC_V \Delta T \quad (74)$$

Stage 3: Isobaric compression, i.e. p_1, V_2, T_2 , compresses to, P_1, V_1, T_3 .

$$W = p_1 \Delta V \quad (75)$$

$$Q = nC_p \Delta T \quad (76)$$

$$\Delta U = nC_p \Delta T - p_1 \Delta V \quad (77)$$

Stage 4: Adiabatic compression, i.e. p_1, V_1, T_3 compresses to, p_2, V_0, T_1 .

$$W = \frac{nR}{\gamma - 1} \Delta T \quad (78)$$

$$Q = 0 \quad (79)$$

$$\Delta U = -\frac{nR}{\gamma - 1} \Delta T \quad (80)$$

I would like to just add a note here. A Carnot cycle is usually built from 2 isotherms and 2 adiabats but the pV diagram I have constructed is from 4 different processes to cover more examples☺.

Efficiency of a Carnot cycle

$$e = \frac{\text{workdone per cycle}}{\text{heat input per cycle}} = \frac{W}{Q_H} \quad (81)$$

For a complete cycle, $\Delta U = Q - W = 0$ hence, $Q = W$. Also known that $Q = Q_H + Q_C = \text{abs}(Q_H) - \text{abs}(Q_C)$.

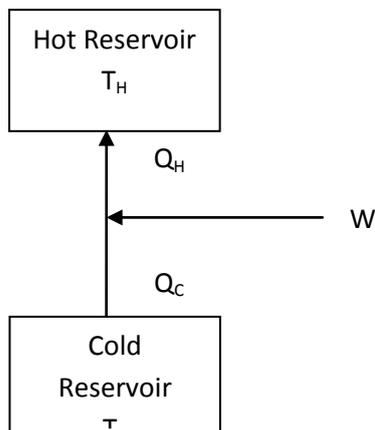
$$e = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \left| \frac{Q_C}{Q_H} \right| \quad (82)$$

This is also equivalent to:

$$e = 1 - \frac{T_C}{T_H} \quad (83)$$

Note that, e , tends towards 1 as T_C tends to zero. USE UNITS OF KELVIN FOR TEMPERATURE!

I also note here that for a Carnot refrigerator it is a little different:



Coefficient of performance:

$$K = \frac{\text{heat extracted}}{\text{work input}} \quad (84)$$

$$K = \frac{|Q_C|}{|Q_H| - |Q_C|} \quad (85)$$

$$K = \frac{T_C}{T_H - T_C} \quad (86)$$

Third law of thermodynamics and Entropy

“It is impossible to reduce the temperature of a body to absolute zero in a finite number of steps”
This law also leads to the next section which is entropy. Entropy, s , is the state variable associated with the order of a system.

Consider the following, an infinitesimal, reversible and isothermal expansion, where $dU = 0$

$$dQ = dW = pdV = \frac{nRTdV}{V} \quad (87)$$

$$\frac{dV}{V} = \frac{1}{nR} \frac{dQ}{T} \quad (88)$$

This leads to the definition of the change in entropy:

$$dS = \frac{dQ}{T} \quad (89)$$

So by integrating we can find a larger change in entropy:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T} \quad (90)$$

So for cyclic processes:

$$\Delta S = 0 \quad (91)$$

Now this leads to a new form of the first law which is used more often:

$$dU = dQ - dW = TdS - pdV \quad (92)$$

Enthalpy and other bits

From the use of equation (92) we form an expression for an internal state variable called the enthalpy, H .

$$\Delta H = Q = dU + pdV \quad (93)$$

This is also equivalent to...

$$\Delta H = nC_p\Delta T \quad (94)$$

So in the case of a reversible, isothermal process we can say that:

$$\Delta S = \frac{Q}{T} = \frac{\Delta H}{T} \quad (95)$$

And in the case of a reversible but non- isothermal process we can say that:

$$\Delta S = \int_1^2 nC_p \frac{dT}{T} \quad (96)$$

And finally in the case of a reversible, adiabatic but isentropic process we find that:

$$Q = 0 \text{ hence } \Delta S = 0 \quad (97 \text{ and } 98)$$

Final notes on Entropy and Boltzmann's equation

For a reversible process we can say that the total change in entropy must be zero and hence we can write that:

$$\Delta S_{system} = \int \frac{dQ}{T} \quad (99)$$

$$\Delta S_{surroundings} = - \int \frac{dQ}{T} \quad (100)$$

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surroundings} = 0 \quad (101)$$

This idea along with our knowledge of reversible processes leads to what is known as the **Clausius Inequality** and with it, 3 consequences:

$$\Delta S \geq \int_a^b \frac{dQ}{T} \quad (102)$$

- For an Isolated system, entropy either increases or stays constant
- The entropy of the universe tends towards a maximum
- The energy content of the universe is a constant (implied by the first law)

Finally, Boltzmann came up with a statistical description for the entropy in terms of his constant K_B and a quantity called the microstate of a system, Ω . I won't go into more detail than just presenting you with the equation:

$$S = K_B \ln(\Omega) \quad (103)$$

Tiny end section on some extra bits (not as vital)

You will discover the deviation and usefulness of this section next year in thermal physics 2. These are used in statistical mechanics; don't be too put off from the word statistical...

Enthalpy:

$$H = U + pV \quad (104)$$

$$dH = dU + pdV + Vdp = dQ + Vdp \quad (105)$$

Gibbs free energy:

$$G = H - TS \quad (106)$$

$$dG = dH - TdS - SdT = Vdp - SdT \quad (107)$$

Helmholtz free energy:

$$F = U - TS \quad (108)$$

$$dF = dU - TdS - SdT = -pdV - SdT \quad (109)$$

And that's about it...hope this guide has been useful, remember use your notes too!

All the best! ☺