①

All Year 2 eqns

PX267

Hamiltonian Mechanics

- $A = \int_{path} \mathcal{L}\, dt$ (action eqn.)

  A - action
  $\mathcal{L}$ - Lagrangian
  dt - w.r.t. time.

- $\mathcal{L} = T - V$ (Classical Lagrangian)

  T - kinetic energy
  V - potential energy

- $\mathcal{L} = -\frac{m_0 c^2}{\gamma} = -m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}}$ (Relativistic Lagrangian)

  m_0 - rest mass
  c^2 - speed of light squared.
  γ - Lorentz factor
  v - velocity

- $T = \frac{1}{2} m\dot{x}^2$ (kinetic energy)

  m - mass
  $\dot{x}$ - velocity

- $V = mgx$ (Gravitational potential energy.)

  m - mass
  g - acceleration due to gravity
  x - height

- $V = \frac{1}{2} kx^2$ (elastic potential energy)

  k - spring constant
  x - displacement

- $V = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$ (electrostatic potential energy)

  Q - charge 1
  q - charge that we move
  ε_0 - permittivity of free space (constant)
  r - distance

- $$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = 0$$ (Euler - Lagrange eqn.)
- $$\vec{F} = \frac{d\vec{p}}{dt} = \dot{\vec{p}}$$ (Newton's 2nd law)

  $\vec{F}$ - vector force
  $\vec{p}$ - vector momentum
- $$H = \dot{q}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \mathcal{L}$$ (Hamiltonian)

  $\dot{q}$ - generalised velocity
  $\mathcal{L}$ - Lagrangian
  H - Hamiltonian.
- $H = T + V$ (Hamiltonian for simple classical systems.)
- $$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = 0$$ (Generalised E-L eqn.)

  (for all i)
- $$H = \sum_i \dot{q}_i \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \mathcal{L}$$ (Generalised Hamiltonian)
- $$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$ (General or canonical momentum)
- $$F_i = \frac{\partial \mathcal{L}}{\partial q_i}$$ (General or canonical force)
- $\vec{F} = -\nabla V$ or $F = -\frac{\partial V}{\partial x}$ (Gradient of potential energy)

  V - potential field
  ∇ - gradient operator.
- $\vec{L} = \vec{r} \times \vec{p}$ (Angular momentum)

  $\vec{r}$ - radial vector
  $\vec{p}$ - linear momentum
  $\vec{L}$ - angular momentum.
- $$F = \frac{mv^2}{r} = m\omega^2 r$$ (Centripetal force)

- $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ (Hamilton's equations)
- $\ddot{x} \propto -x$ (Condition for simple harmonic motion)
- $F_c = mr\dot{\theta}^2$ (Centrifugal force)
  $\dot{\theta}$ - rate of change of angle.
- $M_{ij} = \frac{\partial^2 T}{\partial \dot{q}_i \partial \dot{q}_j} = \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_i} = M_{ji}$ (inertia / mass matrix)
  M_{ij} - entries in inertia matrix, $\underline{\underline{M}}$
  $\underline{\underline{M}}$ is symmetric!
- $K_{ij} = \left.\frac{\partial^2 V}{\partial x_i \partial x_j}\right|_{eq.} = \left.\frac{\partial^2 V}{\partial x_j \partial x_i}\right|_{eq.} = K_{ji}$ (stiffness matrix)
  K_{ij} - entries in stiffness matrix, $\underline{\underline{K}}$.
  $\underline{\underline{K}}$ is symmetric!
  The partial derivatives are evaluated at the equilibrium values.
- $\det(\underline{\underline{K}} - \underline{\underline{M}}\omega^2) = 0$ (Secular equation)
  $\underline{\underline{K}}$ - stiffness matrix
  $\underline{\underline{M}}$ - mass matrix
  ω - angular velocity / frequency.
  det => take determinant.

<u>Environmental Physics</u> PX280

- $W = |Q_H| - |Q_C|$ (Work done by heat engine)
  W - work done
  Q_H - heat energy absorbed
  Q_C - heat energy rejected
- $e = \frac{W}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$ (Efficiency of engine)

- $\Delta U = Q - W$ (1st law of thermodynamics)

ΔU - change in internal energy

Q - heat energy added

W - work done by system

- $dS \geq 0$ (2nd law of thermodynamics)

dS - increase / change in entropy

- $\frac{Q_c}{Q_H} = -\frac{T_c}{T_H}$ (Carnot ratio)

T_c - temperature of cold reservoir

T_H - temperature of hot reservoir

- $e_{Carnot} = 1 - \frac{T_c}{T_H} = \frac{T_H - T_c}{T_H}$ (Carnot efficiency)

- $K_{Carnot} = \frac{T_c}{T_H - T_c}$ (Carnot performance coefficient)

- $\Delta S = \frac{Q}{T}$ (Entropy of reversible isothermal process)

T - absolute temperature

- $\Delta S = \int_1^2 \frac{1}{T} \, dQ$ (Entropy of reversible process)

1 - initial state

2 - final state

dQ - infinitesimal heat into system.

- $\Delta S = 0 = \oint \frac{1}{T} \, dQ$ (Entropy of a reversible cyclic process)

- $\eta = \frac{\text{desired output}}{\text{required input}} = \frac{W}{Q_H} = 1 - \frac{|Q_c|}{Q_H}$ (Efficiency of a power station)

W - work out

Q_H - heat in

Q_c - heat out

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$ (Faraday's law)

ϕ_B - magnetic flux
$\mathcal{E}$ - emf

$\phi_B = BA\cos(\omega t + \phi)$ (Magnetic flux on a wire loop.)

B - magnetic flux density
A - area of loop
ω - angular frequency
ϕ - phase angle

$\mathcal{E} = BAN\omega \sin(\omega t + \phi)$ (emf induced by wire loop.)

N - number of turns of wire.

$\mathcal{E} = I\Delta V = I^2 R_{cable}$ (Energy lost as heat from transmission)

I - current
R_{cable} - resistance of cable
V - voltage

$P = IV$ (Power output at station)

I - current on the grid
V - voltage at station

$$\mathcal{E} = \frac{P^2}{V^2} R_{cable}$$ (Energy loss)

$$R = \frac{\rho L}{A}$$ (Resistance of cable)

ρ - resistivity
L - length of cable
A - cross-sectional area of cable.

$$V_2 = V_1 \frac{N_2}{N_1}$$ (transformer equation)

V_1 - primary voltage
V_2 - secondary voltage
N_1 - primary turns
N_2 - secondary turns

$$\frac{P}{A} = \sigma T^4$$ (Stefan-Boltzmann equation for blackbody)

P - power
A - area
σ - Stefan-Boltzmann constant
T - absolute temperature.

$$P_{sun} = 4\pi R_\odot^2 \sigma T_\odot^4$$ (Power from Sun.)

$R_\odot$ - radius of sun
$T_\odot$ - temperature of sun.

$$v = \sqrt{2gh}$$ (conservation of energy)

v - velocity of water
g - acceleration due to gravity
h - height water falls through.

$$P = \eta \sqrt{2}(gh)^{3/2} A \rho$$ (Power of hydroelectricity)

η - efficiency
A - area of pipe section
ρ - density of water

$$U = \frac{g \rho S R^2}{2}$$ (potential energy for tidal power)

ρ - density of tidal water
S - surface area of trapped water
R - head height

$$T = \frac{\rho_{air} A \delta t v^3}{2}$$ (kinetic energy of wind)

A - area of turbine
δt - small time
v - velocity of wind

$$P = \frac{T}{t} = \frac{\rho_{air} A v^3}{2}$$ (Power of wind)

t - time
P - power

- $P = \frac{\rho}{2} A v (v_1^2 - v_2^2)$ (Betz law relationship)

P - power absorbed
A - area of turbine
v - windspeed at turbine
v_1 - downstream windspeed
v_2 - upstream windspeed.

- $F = \frac{dp}{dt} = \rho A v (v_1 - v_2)$ (Force on turbine)

p - momentum

- $P = Fv = \rho A v^2 (v_1 - v_2)$ (Power on turbine)

F - force
v - windspeed

- $v = \frac{v_1 + v_2}{2}$ (velocity relationship.)

- $P = \frac{\rho}{2} A v_1^3 C_p(x)$ (Real performance)

$C_p(x) = \frac{1}{2}\left(1 + \frac{v_2}{v_1}\right)\left(1 - \frac{v_2^2}{v_1^2}\right)$

C_p - power coefficient

- $\langle P \rangle = C_f C_p \frac{\rho_{air}}{2} A V_{rated}^3$ (Average power output)

C_f - capacity factor
C_p - power coefficient
V_{rated} - rated wind velocity

- $V_1 = V_0 \left(\frac{H_1}{H_0}\right)^\alpha$ (Windspeed comparison)

V_0 - velocity at H_0
V_1 - velocity at H_1
α - factor depending on terrain
H_0 - initial height
H_1 - final height

- $\Delta M c^2 = Z m_p c^2 + (A-Z) m_n c^2 - M(^A_Z X) c^2$ (Mass defect)

Δm - mass defect
c - speed of light
Z - atomic ~~mas~~ number
A - atomic mass
m_p - mass of proton
m_n - mass of neutron
$M(^A_Z X)$ - mass of element X.

- $$D(R) = \frac{D(0)}{1 + \exp\left(\frac{R-b}{a}\right)}$$ (charge / mass density)

$D(0)$, a, b are constants (see notes)
R - radius

- $$P = \frac{N\sigma}{A}$$ (Reaction probability)

N - number of nuclei
σ - Reaction cross-section
A - area of target

- $\sigma_g = \pi R^2$ (geometrical cross-section)

R - radius of atom

- $\sigma_a = \sigma_{in} + \sigma_{\gamma} + \sigma_f$ (Neutron absorption)

σ_a - absorption cross-section
σ_{in} - inelastic scattering cross-section
σ_γ - neutron capture cross-section
σ_f - fission cross-section

- $\sigma_{total} = \sigma_{el} + \sigma_a \geq \sigma_g$ (total cross-section)

σ_{el} - elastic scattering cross-section

- $$n = \frac{\rho N_A}{M}$$ (number density)

ρ - density
N_A - ~~as~~ Avogadro's number
M - molar mass

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- $\lambda = \frac{1}{n\sigma}$ (mean free path)

n - number density
σ - cross-section
λ - mean free path

- $d = \sqrt{N_c}\,\lambda$ (distance travelled)

N_c - number of collisions

- $R = n_n v_{th,n} n_u \sigma_f$ (Reaction rate)

n_n - number density of neutrons
$v_{th,n}$ - thermal velocity of neutrons
n_u - number density of U-235
σ_f - fission cross-section

- $\frac{1}{2} m v_{th}^2 = k_B T$ (thermal velocity)

m - mass of particle
k_B - Boltzmann constant

- $\frac{d\phi}{dt} = \frac{(K-1)\phi}{\tau} = \frac{\phi}{T_R}$ (neutron equation)

$\phi(t)$ - number of neutrons at time, t.
K - some factor
τ - mean neutron lifetime

- $T_R = \frac{\tau}{K-1}$ (Reactor period)

- $K = 1$ - steady output power (K - value)

- $\phi(t) = \phi(t_0) \exp\left(\frac{t}{T_R}\right)$ (no. of neutrons)

$\phi(t_0)$ - number of neutrons at t_0.

- $\tau_{avg} = (1-\alpha)\tau_p + \alpha \tau_d$ (mean lifetime of neutron)

τ_p - prompt time
τ_d - delayed time
α - percentage of delayed neutrons

$W = 3k_B T n$ (energy density of plasma)

T - temperature

n - number density

$P_L = \frac{W}{\tau_E} = \frac{3k_B T n}{\tau_E}$ (Power loss rate)

τ_E - confinement time

$f = \frac{1}{4} n^2 \langle \sigma v \rangle$ (number of collisions of D+T)

σ - cross-section for fusion

v - relative velocity of D+T.

$P_H = f E_\alpha = \frac{1}{4} n^2 E_\alpha \langle \sigma v \rangle$ (Power input rate)

E_α - energy injected by a particle.

$n T \tau_E \geq \frac{3k_B}{\beta}$ (Lawson Criterion)

β - constant of proportionality

T - temperature

$\lambda = 8.5 \times 10^{21} \frac{T^2}{n}$ (Ion mean free path)

T - temperature

n - number density

$D_r = \frac{\delta^2}{\tau}$ (Diffusive transport)

δ - ~~dif~~ step length

D_r - diffusion coefficient

τ - time between collisions

$\vec{u}_{E\times B} = \frac{\vec{E} \times \vec{B}}{B^2}$ (E cross B drift)

$\vec{u}_{E\times B}$ - drift velocity

$\vec{E}$ - E-field

$\vec{B}$ - B-field

- $(1-a)(S \cdot \pi R^2) = \sigma T^4 \cdot (4\pi R^2)$ (Simplest climate model)

a - albedo
S - Solar constant
R - radius of Earth
T - temperature of Earth
σ - stefan - Boltzmann constant.

- $I_E \, dE = \frac{2\pi}{h^3} \frac{E^3}{c^2} \frac{1}{e^{E/kT} - 1} dE$ (Planck distribution)

E - energy
h - Planck's constant
c - speed of light
k - Boltzmann's constant
T - absolute temperature

- $E = \hbar\omega = hf = \frac{hc}{\lambda}$ (Photon energy)

ħ - Reduced Planck's constant
ω - angular frequency
f - frequency
λ - wavelength

- $W(\omega)\, d\omega = \frac{\omega^3}{c^3 \pi^2} \frac{\hbar}{e^{\hbar\omega/kT} - 1} d\omega$ (Energy density)

- $I_\lambda \, d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$ (Planck distribution again)

- $\frac{dP_k(t)}{dt} = B_{ik} W(\omega)$ (Probability of system in excited state)

B_{ik} - Einstein coefficient
$W(\omega)$ - energy density
$P_k(t)$ - Probability
(Describes how it varies with time.)

- $B_{1k} = \frac{\pi}{3\varepsilon_0 \hbar^2} |\mu_{k1}|^2$ (Einstein coefficient)

$|\mu_{k1}|^2$ - transition dipole moment
ε_0 - permittivity of free space.

- $\frac{dN_1}{dt} = -B_{12} W(\omega) N_1 + B_{21} W(\omega) N_2 + A_{21} N_2$ (2 state system)

N_1 - population / number in ground state
N_2 - number in excited state
A_{21} - Einstein coefficient for spontaneous emission

- $I(z) = I(0) \exp(-kz)$ (Beer - Lambert law)

$I(z)$ - intensity at z
$I(0)$ - initial intensity
k - some function of ω.

- $B_{12} = \frac{Vn}{N_1 \hbar} \int_{band} \frac{k(\omega)}{\omega} d\omega$ (measuring $k(\omega)$)

V - volume
n - refractive index
(integrated over a band of angular frequencies)

- $I(l^0) = I(0) \cdot 10^{-OD}$ (Recast Beer - Lambert law)

l_0 - Path length &
OD - optical density

- $k = \frac{OD}{l^0} \cdot \frac{1}{\log_{10} e}$ (k term from Beer - Lambert law)

- $OD = \varepsilon l C$ (absorption in plants)

ε - Molar extinction coefficient
C - concentration
l - path length

- $\sigma = \pi r^2 P$ (Absorption cross - section)

σ - absorption cross - section
r - radius of particle
P - probability of photon being absorbed.

- $dp = -g\rho\, dz$ (Hydrostatic equation)

p - pressure
ρ - density
g - gravity acceleration

- $pV = nRT$ (Ideal gas.)

n - number of moles
R - molar gas constant

- $$\text{Mass } (m) \underset{\text{multiply by molar mass}}{\overset{\text{divide by molar mass}}{\Leftrightarrow}} \text{Moles } (n) \underset{\text{multiply by } N_A}{\overset{\text{divide by } N_A}{\Leftrightarrow}} \text{Number of particles } (N)$$

N_A - Avogadro's Number

- $H_e = \dfrac{R' T_{av}}{g} \simeq 7.3\text{ km}$ (Effective scale height)

R' - specific gas constant ($R' = R/M$ (M - molar mass))
T_{av} - average temperature
g - gravity

- $p = P_0 \exp(-z/H_e)$ (Pressure varying with altitude)

P_0 - ground pressure
z - altitude

- $\Gamma_d = -\dfrac{g}{c_p}$ (Dry adiabatic lapse rate)

c_p - Heat capacity at constant pressure

- $\Gamma_s = \Gamma_d + \dfrac{\Delta H_{vap}}{c_p}\dfrac{\partial w}{\partial z}$ (Saturated adiabatic lapse rate)

ΔH_{vap} - vaporization enthalpy
w - mass fraction of water vapour.

- $(1-a)\,S/4 = \sigma T_a^4 + b\sigma T_s^4$ (Radiation balance)

a - albedo ($\simeq 0.3$)
T_a - temperature of surface

T_s - temperature of surface
S - solar constant
σ - Stefan - Boltzmann constant
t - transmission constant ($\simeq 0.06$)

- $\Delta T_s = \left(\frac{\partial I}{\partial T_s} \right)^{-1} \Delta I = G \Delta I$ (Radiative forcing)

ΔT_s - change in temperature
G - gain factor, equal to one over $\partial I / \partial T_s$ (~ 0.3)
ΔI - radiative forcing

- $\Delta T_s = G_F \Delta I$ (Climate sensitivity)

G_F - Climate sensitivity parameter ($\sim$ 0.8 K/Wm^{-2})

- $\frac{d\vec{u}}{dt} \rho \, d\tau = \vec{F}_{pres} + \vec{F}_{viscous} + \vec{F}_{Coriolis} + \vec{F}_{gravity}$ (Motion of air)

$\vec{u}$ - velocity vector
ρ - density
$d\tau$ - volume element ($dx\,dy\,dz$)

- $\vec{F}_{pres} = -\nabla p \, d\tau$
- $\vec{F}_{viscous} = \mu \frac{\partial^2 u_x}{\partial z^2} d\tau$
- $\vec{F}_{Coriolis} = -2 \vec{\Omega} \times \vec{u} \rho \, d\tau$
- $\vec{F}_{gravity} = \vec{g} \rho \, d\tau$

(4 forces in motion of air equation)

μ - dynamic viscosity
$\vec{\Omega}$ - Earth's rotation vector
$\vec{u}$ - velocity in rotating frame
$\vec{g}$ - gravitational field vector

- $-\frac{1}{\rho} \nabla p = 2 (\vec{\Omega} \times \vec{u})$ (Geostrophic flow)

- $\frac{|\nabla p|}{\rho f} = v$ (Speed of flow)

f - Coriolis parameter ($f = 2 \Omega \sin \beta$)
β - latitude

- $$\frac{\partial}{\partial z}\left(\frac{\hat{k} \times \vec{u}}{T}\right) = \frac{-g}{f T^2} \nabla T$$ (Coupling horizontal and vertical properties of atmosphere)

f - Coriolis parameter
$\hat{k}$ - unit vector in $\hat{k}$-direction
$\vec{u}$ - velocity in horizontal direction

- $$\frac{du_x}{dt} = (\vec{u} \cdot \nabla) u_x + \frac{\partial u_x}{\partial t}$$ (x-component of $d\vec{u}/dt$)

u_x - x-component of $\vec{u}$

<u>Physics of fluids</u> PX264

- $$\tau = \frac{F}{A}$$ (Stress)

F - force
A - area
τ - stress

- $$e = \frac{\partial x}{\partial y}$$ (Strain)

- $$\tau = Ge$$ (Shear modulus)

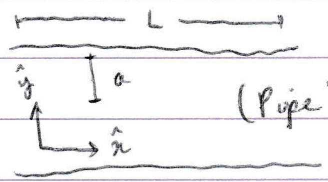
G - shear modulus

- $$\tau = \mu \frac{de}{dt} = \mu \frac{\partial v_x}{\partial y}$$ (Fluid stress)

μ - viscosity
v_x - fluid flow velocity

- $$v_x(y) = \frac{Q}{2\mu}(a^2 - y^2)$$ (laminar, viscous, steady flow)

Q - some constant
a - radius of pipe
μ - viscosity of fluid
y - height in pipe.



- $v_z(r) = \frac{Q}{4\mu}(a^2 - r^2)$ (Poiseuille flow)

a - radius of pipe
r - radial distance
Q - some constant given by boundary conditions

- $\nabla p = \mu \nabla^2 \vec{u}$ (Steady state, viscous flow)

p - pressure
$\vec{u}$ - velocity

- $\nu = \frac{\mu}{\rho}$ (Kinematic viscosity)

ρ - density

- $Re = \frac{\rho_0 v_0 L_0}{\mu}$ (Reynolds number)

ρ_0 - typical mass density
v_0 - typical speed
L_0 - typical length scale

- $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$ (Continuity equation)

- $\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \mu \nabla^2 \vec{v} + \frac{\mu}{3} \nabla(\nabla \cdot \vec{v}) - \rho g \vec{k}$

$\vec{v}$ - velocity (Navier-Stokes equation)
g - gravity acceleration
$\vec{k}$ - direction of gravity (radially inwards, hence minus sign)

Also written as:

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \mu \nabla^2 \vec{v} + \frac{\mu}{3} \nabla(\nabla \cdot \vec{v}) + \rho \vec{g}$$

and $\frac{d\vec{v}}{dt}$ is the advective derivative, $\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}$

$\vec{g}$ is gravity vector, $-g\hat{k}$

- $\frac{d}{dt}\left(\frac{P}{\rho^{\gamma}}\right) = 0$ (Adiabatic fluid)

γ - ratio of specific heats | adiabatic ratio
P - pressure
ρ - density

- $\rho = \text{constant}$ and $\nabla \cdot \vec{v} = 0$ (incompressible flow)
- $\frac{\delta}{d} \simeq \frac{1}{\sqrt{Re}}$ (Boundary layer thickness)

δ - boundary layer thickness
d - length of plate

- $P + \frac{1}{2}\rho v^2 + \rho g z = \text{constant along streamline}$ (Bernoulli's equation)

z - height
(Only applies for incompressible, steady, inviscid flow.)

- $c_s = \sqrt{\frac{\gamma P_0}{\rho_0}}$ (Sound speed)

c_s - speed of sound
γ - adiabatic ratio

- $K = \oint_{\Gamma} \vec{v} \cdot d\vec{l}$ (Circulation around loop)

Γ - loop we integrate around
$d\vec{l}$ - line element
K - circulation

- $\vec{\omega} = \nabla \times \vec{v}$ (Vorticity)

$\vec{\omega}$ - vorticity vector

- $\frac{dK}{dt} = \frac{d}{dt}\left[\oint_{\Gamma} \vec{v} \cdot d\vec{l}\right] = 0$ (Kelvin circulation theorem)
- $\nabla \times \vec{v} = \vec{\omega} = 0$ (Irrotational flow)
- $\nabla \phi = \vec{v}$ (Potential flow)

(This is only true for irrotational flow, ie. $\nabla \times \vec{v} = 0$)
ϕ - Scalar field / flow potential

- $\vec{v} = \frac{q}{2\pi r}\hat{r}$ (2D point source)

q - flow rate

- $v_x = \frac{\partial \psi}{\partial y}$ and $v_y = -\frac{\partial \psi}{\partial x}$ (Streamfunctions)

ψ - streamfunction

(ψ = constant gives us streamlines)

- $\frac{\partial \phi}{\partial t} + \frac{v^2}{2} + \frac{P}{\rho} + gz$ = constant everywhere (Generalised Bernoulli)

ϕ - flow potential

(This is for irrotational, incompressible and inviscid flow)

- $\vec{L} = \rho(\vec{k} \times \vec{v})$ (Magnus effect)

$\vec{L}$ - lift vector

$\vec{k}$ - circulation vector

- $F = b\frac{\rho v^2}{2}A$ (drag force)

b - drag coefficient

A - cross-sectional area

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Thermal Physics

- $\sum_i P_i = 1$ (Normalisation)
- $\langle x \rangle = \sum_i x_i P_i = \frac{1}{n} \sum_{i=1}^{N} x_i$ (mean)
- $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ (Standard deviation)

x - some random variable
σ_x^2 - variance
σ_x - standard deviation
$\langle x^2 \rangle$ - mean of x^2 terms
$\langle x \rangle^2$ - mean squared

- $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$ (Bayes theorem)

$P(A|B)$ - probability of A given B occurred
$P(A)$ - probability of A occurring
$P(B)$ - probability of B occurring

- $dU = đQ + đW = T\,dS - p\,dV$ (1^{st} law of thermo)

dU - internal energy
$đQ$ - change in heat energy } inexact differentials
$đW$ - change in work done
T - absolute temp
dS - change in entropy
p - pressure
dV - change in volume.

- $V = \frac{1}{2} m\omega^2 x^2$ (Potential energy of SHO)

ω - angular velocity of oscillator
x - displacement
m - mass of object

- $E_n = (n + \frac{1}{2})\hbar\omega$ (Energy of SHO)

n - quantum number ($n \in 1, 2, 3, \ldots$)
$\hbar$ - reduced Planck

- $S = k_B \ln \Omega$ (Entropy)

k_B - Boltzmann's constant
Ω - number of microstates
S - entropy

- $\Omega_{total} = \Omega_1 \cdot \Omega_2 \cdot \Omega_3 \cdot \ldots \cdot \Omega_N$ (combining microstates)
- $pV = nRT = Nk_BT$ (ideal gas equation)

p - pressure
V - volume
n - no. of moles
R - molar gas constant
T - absolute temperature
N - no. of molecules
k_B - Boltzmann again

- $\frac{dS}{dE} = \frac{d(k_B \ln \Omega)}{dE} = \frac{1}{T}$ (temperature of isolated system)
- $\beta = \frac{1}{k_BT}$ (useful)
- $P(E_i) = \frac{e^{-\beta E_i}}{Z}$ (Boltzmann distribution)

$P(E_i)$ - probability of system having energy E_i.

- $Z = \sum_i e^{-\beta E_i}$ (Partition function)
- $S = -k_B \sum_i P_i \ln P_i$ (Gibbs's entropy formula)
- $\langle E \rangle = \frac{n}{2} k_BT$ (Equipartition theorem)

n - number of quadratic, independent degrees of freedom a system has.

- $C_v = \left(\frac{\partial \langle E \rangle}{\partial T}\right)_{\text{fixed } v}$ (Heat capacity)

C_v - specific heat capacity

- $F = U - TS$ (Helmholtz free energy)

F - H. free energy
U - internal energy
T - temperature
S - entropy

- $H = U + pV$ (Enthalpy)

H - enthalpy
p - pressure
V - volume

- $G = H - TS$ (Gibbs free energy)

G - Gibbs free energy

- $U = -\dfrac{d \ln Z}{d\beta}$ (internal energy)

Z - partition function

- $S = k_B(\beta U + \ln Z)$ (Entropy)
- $F = U - TS = -k_B T \ln Z$ (Helmholtz free energy)
- $a\sum_{n=1}^{\infty} r^n = \dfrac{a}{1-r}$ (Geometric sum to infinity)

- $S' = \hbar\sqrt{s(s+1)}$ (Spin angular momentum)

S' - angular momentum intrinsic to particle
S - spin quantum number ($s \in 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$)

- $S'_z = \hbar m_s$ (component of spin vector)

m_s - magnetic spin quantum number ($m_s \in -s, -s+1, \ldots, s-1, s$)

- $Z = \sum_i g(E_i) e^{-\beta E_i}$ (Degenerate partition function)

$g(E_i)$ - number of distinct states with energy E_i

- $Z_{total} = Z_a \cdot Z_b \cdot Z_c \cdot \ldots \cdot Z_n$ (Combining partition functions)
- $Z_N = (Z_1)^N$ (for spin $\frac{1}{2}$ paramagnet)
- $dU = T\,dS - m\,dB$ (magnetic systems)

m - magnetic moment

- $dH \leq 0$ for fixed S and P
- $dG \leq 0$ for fixed T and P
- $dF \leq 0$ for fixed T and V
- $\sigma_E = T\sqrt{k_B C_V}$ (standard deviation of energy)
- $g(k)\,dk = \dfrac{Vk^2}{2\pi^2}\,dk$ (Density of States)

V - volume of container

k - wavenumber

- $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$ (Standard integral)

α - constant

- $n_Q = \left(\dfrac{mk_BT}{2\pi\hbar^2}\right)^{3/2}$ (Quantum concentration)

m - mass of particle ($n_Q = \dfrac{1}{\lambda_{th}^3}$)

n_Q - quantum concentration

- $\lambda_{th} = \dfrac{h}{\sqrt{2\pi m k_B T}}$ (Thermal wavelength)

λ_{th} - thermal wavelength

- $Z_N = \dfrac{Z_1^N}{N!}$ (Indistinguishability)

N - number of particles

- $\ln(m!) \simeq m\ln m - m$ (Stirling's approximation)
- $S = \dfrac{U-F}{T} = Nk_B\left[\dfrac{5}{2} - \ln\left(\dfrac{N\lambda_{th}^3}{V}\right)\right]$ (Sackur-Tetrode equation)

S - entropy

U - internal energy

F - Helmholtz free energy

- $\mu = \left(\dfrac{\partial U}{\partial N}\right)_{S,V} = \dfrac{G}{N}$ (Chemical potential)

G - Gibb's free energy

μ - Chemical potential

- $P_i = \dfrac{e^{-\beta(E_i - \mu N_i)}}{\mathcal{Z}}$ (Gibbs distribution)
- $\mathcal{Z} = \sum_i e^{-\beta(E_i - \mu N_i)}$ (Grand partition function)

$\mathcal{Z}$ - grand partition function
E_i - energy of state i
P_i - probability of state i
N_i - number of particles in state i

- $\Phi_G = -k_B T \ln \mathcal{Z}$ (Grand potential)

Φ_G - grand potential

- $u_\lambda = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{e^{hc/\lambda k_B T} - 1}$ (Planck distribution law)

u_λ - spectral energy density
λ - wavelength of EM radiation

- $\omega = ck$ (photon relation)

c - speed of light
k - wave vector
ω - angular frequency

- $I = \sigma T^4$ (Stefan-Boltzmann law)

I - intensity

- $\lambda_{max} T = 2.897 \text{ mm K}$ (Wien displacement law)

λ_{max} - maximum wavelength

- $\omega_D = \left(\dfrac{6N\pi^2 v_s^3}{V} \right)^{1/3}$ (Debye frequency)

ω_D - Debye frequency
v_s - speed of sound
V - volume
N - no. of particles

- $\Theta_D = \dfrac{\hbar \omega_D}{k_B}$ (Debye temperature)

- $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{\frac{2}{3}}$ (Fermi energy)

N - number of carriers
V - volume of metal

- $T_F = \frac{E_F}{k_B}$ (Fermi temperature)
- $f_{FD}(E) = \frac{1}{e^{\beta(E-\mu)}+1}$ (Fermi-Dirac distribution)
- $f_{BE}(E) = \frac{1}{e^{\beta(E-\mu)}-1}$ (Bose-Einstein distribution)
- $N = \int_0^\infty g(E) f(E)\, dE$ (Number of total particles)
- $U = \int_0^\infty E\, g(E) f(E)\, dE$ (Internal energy)

$f(E)$ - distribution function (f_{FD} for fermions, f_{BE} for bosons)
$g(E)$ - density of states

- $\langle n \rangle = -\frac{1}{\beta}\frac{\partial \ln \mathcal{Z}}{\partial E}$ (Mean occupation number)

$\mathcal{Z}$ - grand partition function
$\langle n \rangle$ - mean occupation number

- $\int_0^\infty (2S+1)\frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\frac{E^{\frac{1}{2}}}{e^{\beta(E-\mu)}-1}\, dE$ (General 3D boson gas)

S - spin states

- $k_B T_c = \frac{2\pi\hbar^2}{m}\left(\frac{n}{2.612}\right)^{\frac{2}{3}}$ (Critical temperature for BEC)

n - carrier density, $n = N/V$.

Quantum Mechanics

- $E_k = h\nu - \phi$ (Photoelectric equation)

  E_k - kinetic energy of electrons
  h - Planck's constant
  ν - photon frequency
  ϕ - work function

- $h\nu = \frac{hc}{\lambda}$ (Photon energy)

  c - speed of light
  λ - photon wavelength

- $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$ (Compton scattering)

  λ' - wavelength of scattered radiation
  λ - wavelength of incident radiation
  m - mass of electron (rest mass)
  ϕ - scattering angle

- $I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)}$ (Planck radiation law)

  $I(\lambda)$ - spectral emittance of blackbody
  λ - wavelength
  k - Boltzmann constant
  T - absolute temperature of blackbody

- $\lambda = \frac{h}{p} = \frac{h}{mv}$ (de Broglie wavelength)

  λ - de Broglie wavelength
  p - momentum
  v - velocity

- $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ (kinetic energy for non-relativistic particle)

  m - mass of particle
  p - momentum of particle

- $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

  (TDSE) (1-dimension)

  $\hbar$ - reduced Planck's constant
  Ψ - time-dependant wavefunction
  V - potential energy
  i - imaginary unit.

- $\hbar = \frac{h}{2\pi}$ (Reduced Planck's constant.)

- $f(t) = \exp\left(\frac{-iE\hbar}{t}\right)$ (time evolution of stationary states)

  E - energy
  t - time

- $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x)}{\partial x^2} + V(x)\Psi(x) = E\,\Psi(x)$$

  (TISE) (1-dimension)

  $\Psi(x)$ - time-independant wavefunction
  E - energy eigenvalue

- $\Delta x \Delta p \geq \hbar/2$ (H.U.P. - position and momentum ~~time~~)

  Δx - position uncertainty
  Δp - momentum uncertainty

- $\Delta t \Delta E \geq \hbar/2$ (H.U.P. - energy and time)

  Δt - time uncertainty
  ΔE - energy uncertainty

- $E = \hbar\omega$ (energy/angular velocity relation)

  ω - angular velocity

- $p = \hbar k$ (momentum/wavenumber relation)

  k - wavenumber

- $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ (energy eigenvalues for particle in infinite potential well.)

  n - energy level, $n \in 1, 2, 3, \ldots$
  L - length of box
  m - mass of particle.

- $V(x) = \frac{1}{2}k'x^2$ (potential of harmonic oscillator)
  k' - 'spring constant' (curvature of potential well.)
- $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ (finite well wavefunction)
  $n \in 1, 2, 3, \ldots$
- $E_n = (n + \frac{1}{2})\hbar\omega$ (energy eigenvalues for harmonic oscillator)
  $n \in 0, 1, 2, \ldots$
- $\hat{H}\psi_n = E_n \psi_n$ (TISE as eigenvalue equation)
  $\hat{H}$ - Hamiltonian operator
  ψ_n - eigenstate / eigenfunction
  E_n - eigenvalue, energy.
- $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, dx = 1$ (normalisation condition) (1 dimension)
- $\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} |\Psi(r,\theta,\phi)|^2 \sin\theta \, r^2 \, dr \, d\theta \, d\phi = 1$
  (normalisation condition (3D spherical polars))
  θ - polar angle.
  ϕ - azimuthal angle.
  r - radial component
- $\vec{L} = \vec{r} \times \vec{p}$ (Angular momentum, classical)
  $\vec{L}$ - angular momentum
  $\vec{r}$ - position vector
  $\vec{p}$ - linear momentum vector
- $L_z = x p_y - y p_x$ (z-component of angular momentum)
  x - component in $\hat{x}$ of $\vec{r}$
  p_y - component in $\hat{y}$ of $\vec{p}$
- $-\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$ (TISE) (3D cartesians)
  ∇^2 - Laplacian operator
- $L_z < |\vec{L}|$ (H.U.P. relation)
  L_z - z component of angular momentum
  $|\vec{L}|$ - modulus of angular momentum

- $n \in 1, 2, 3, \dots$ (Principal quantum number)
- $l \in 0, 1, 2, \dots, n-1$ (Orbital quantum number)
- $m_l \in -l, \dots, 0, \dots, +l$ (Magnetic quantum number)
- ~~Total~~ $L = \hbar\sqrt{l(l+1)}$ (Total angular momentum)

  L - total angular momentum.
- $L_z = \hbar m_l$ (z-component of angular momentum)

  m_l - magnetic quantum number
- $\vec{m} = \pi r^2 I \hat{z}$ (Magnetic moment)

  r - radius of loop

  I - current

  $\hat{z}$ - z-direction
- $\Delta E = 2|\vec{m}| B_z$ (Zeeman effect)

  $|\vec{m}|$ - magnitude of magnetic moment

  B_z - z-component of magnetic field
- $V(r) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze}{r^2}$ (Coulomb potential)

  ε_0 - permittivity of free space

  Z - proton number

  e - charge of electron

  r - radial distance
- $S' = \hbar\sqrt{s(s+1)}$ (Spin angular momentum)

  S' - spin angular momentum

  s - spin number, $\frac{1}{2}$.
- $\Psi = \sum_n c_n \psi_n$ (superposition of eigenfunctions)

  Ψ - wavefunction

  ψ_n - eigenfunction

  c_n - superposition coefficient
- $\hat{H}\Psi = i\hbar \dfrac{\partial \Psi}{\partial t}$ (TDSE) (3 dimensional eigenvalue equation)

  $\hat{H}$ - Hamiltonian operator

- $\langle q \rangle = \int_E \Psi^* \hat{Q} \Psi \, d\tau$ (expectation value)

$\langle q \rangle$ - expectation value of $\hat{Q}$
$\hat{Q}$ - operator giving an observable
Ψ - eigenstate
integrated over many experiments.

- $\int_E \Psi_i^* \Psi_j \, d\tau = \langle \Psi_i | \Psi_j \rangle = \delta_{ij}$ (orthonormality)

$\langle \Psi_1 | \Psi_2 \rangle$ - inner product

- $\delta_{ij} = 1$ if $i = j$ (Kronecker Delta)
$\delta_{ij} = 0$ if $i \neq j$
i, j are indices

- $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$ (momentum operator)(1 dimension)

- $\hat{p} = -i\hbar \nabla$ (momentum operator)(3D)
- $\hat{x} = x$ (position operator)(1D)
- $\hat{r} = \vec{r}$ (position operator)(3D)
- $\delta(x - x') = 0$ (Dirac Delta function)
for $x \neq x'$
- $\sigma_q^2 = \langle q^2 \rangle - \langle q \rangle^2$ (Variance)

$\langle q^2 \rangle$ - mean value of squared terms
$\langle q \rangle^2$ - mean value, squared
σ_q^2 = variance

- $\langle q \rangle = \frac{1}{N} \sum_N q_n$ (Mean)

N - number of values

- $\int_E f^* \hat{Q} g \, d\tau = \int_E g \hat{Q}^* f^* \, d\tau$ (Hermiticity)
- $\hat{Q}^* f^* = (\hat{Q} f)^*$ (Conjugates)
$*$ means take complex conjugate.

- $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ (commutators)
  if $[\hat{A}, \hat{B}] = 0$, the operators commute.
- $\sigma_A \sigma_B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$ (Generalised H.U.P.)

  σ_A - uncertainty in observable A.
  σ_B - uncertainty in observable B.
- $\frac{d\hat{Q}}{dt} = i/\hbar \langle [\hat{H}, \hat{Q}] \rangle$ (Ehrenfest theorem)
  here, the big angle brackets mean the expectation value of the commutators.
  $\hat{H}$ - Hamiltonian operator.
- $\hat{L}_z = -i\hbar \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$ (z-component of angular momentum operator) (Cartesian)
- $\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$ (z-component of angular momentum operator) (spherical polars)
- $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ (angular momentum operator squared)
- $[\hat{L}_x, \hat{L}_y] = i\hbar L_z$ (angular momentum commutators)
- $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$ (raising and lowering operators)
- $[\hat{L}_z, \hat{L}_\pm] = \pm \hbar \hat{L}_\pm$ (commutators)
- $[\hat{L}^2, \hat{L}_\pm] = 0$ (commutators)
- $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ (Orthonormality)
- $\int \psi_1^* \psi_2 \, d\tau = \langle \psi_1 | \psi_2 \rangle$ (Bra - ket)
- $\langle q^2 \rangle = \langle \psi | \hat{Q}\hat{Q} | \psi \rangle$ (Variance again)
- $E_n = -\frac{13.6 \text{ eV}}{n^2}$ (Hydrogen atom energy levels)

  n - quantum number, $n \in 1, 2, 3, \ldots$
- $\psi_A(\vec{r}_1, \sigma_1, \vec{r}_2, \sigma_2) = -\psi(\vec{r}_2, \sigma_2, \vec{r}_1, \sigma_1)$ (Exchange symmetry)
  σ - spin
- $\psi(x, y, z) = Ae^{i\vec{k}\cdot\vec{r}}$ (3D particle in a box wavefunction)
  $\vec{k}$ - wavevector
  $\vec{r} = \vec{r}(x, y, z)$

$$n(E) = \frac{V}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$$ (Density of states)

V - volume

m - mass of particle

$$E_{total} = \int_0^{E_F} E\, n(E)\, dE$$ (total energy)

E_F - Fermi energy

$$v_F = \frac{\hbar k_F}{m}$$ (Fermi velocity)

k_F - Fermi wavevector / number

$$E = \frac{\hbar^2 k^2}{2m}$$ (Energy in terms of wavevector)

$$F(E,\mu,T) = \frac{1}{e^{(E-\mu)/k_B T}+1}$$ (Fermi - Dirac distribution)

E - energy

μ - chemical potential

T - absolute temperature

k_B - Boltzmann

$$\hat{\mu}_s = -\frac{eg}{2m}\hat{S}_z$$ (Magnetic moment)

e - charge on electron

g - factor

$\hat{S}_z$ - spin operator (eigenvalues of $\pm\frac{1}{2}\hbar$)

$$M = \mu_B(N_\downarrow - N_\uparrow) \simeq \mu_B\, n(E_F)\, \mu_B B$$ (Magnetisation)

$N_\uparrow$ - number in spin up state

$N_\downarrow$ - number in spin down state

μ_B - Bohr magneton

$$\chi = \frac{dM}{dB} = \mu_B^2\, n(E_F)$$ (Susceptibility)

M - magnetisation

χ - susceptibility

- $\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$ (Current density, analogy to conservation of mass in fluids)

$\vec{j}$ - current density vector

ρ - density

- $\vec{j} = \frac{\hbar}{2im}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*)$ (Solution to current density) (3D)

Ψ - wavefunction

- $\vec{R} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$ (Lattice vector)

n_1, n_2, n_3 span over all integers

$\vec{a}, \vec{b}, \vec{c}$ are 3 non-coplanar vectors $\Rightarrow$ primitive vectors

- $V_c = \vec{a} \cdot (\vec{b} \times \vec{c})$ (Unit cell of lattice)

V_c - volume of parallelepiped $\equiv$ unit cell

- $\vec{G}_{h,k,l} = h\vec{A} + k\vec{B} + l\vec{C}$ (Reciprocal vector)

$\vec{A}, \vec{B}, \vec{C}$ are primitive vector for reciprocal lattice

h, k, l are integers too.

- $e^{i(\vec{G}_{h,k,l} \cdot \vec{R}_{n_1,n_2,n_3})} = 1$ (Condition for reciprocal lattice)

$$\left.\begin{aligned} \vec{A} &= \frac{2\pi(\vec{b} \times \vec{c})}{\vec{a} \cdot (\vec{b} \times \vec{c})} \\ \vec{B} &= \frac{2\pi(\vec{c} \times \vec{a})}{\vec{a} \cdot (\vec{b} \times \vec{c})} \\ \vec{C} &= \frac{2\pi(\vec{a} \times \vec{b})}{\vec{a} \cdot (\vec{b} \times \vec{c})} \end{aligned}\right\} \text{(Reciprocal lattice primitive vector)}$$

- $\Psi(x) = e^{ikx} u(x)$ (1D Bloch's theorem)

$u(x) = u(x+a) = u(x+2a) = \ldots$

Can be converted to 3D by $x \to \vec{r}$ and $k \to \vec{k}$

- $N = \int_0^\infty F(E,\mu,T)\, n(E)\, dE$ (No. of electrons)

$F(E,\mu,T)$ - Fermi-Dirac distribution

$n(E)$ - density of states

- $I = I_s\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$ (Shockley diode equation)

e - electron charge
V - voltage across diode
T - temperature of diode
I_s - saturation current

- $\hat{L}^2 Y_{l,m}(\theta, \phi) = l(l+1)\hbar^2 Y_{l,m}(\theta, \phi)$ (Angular moment. eigenvalue eqn)

$\hat{L}^2$ - operator
$Y_{l,m}$ - eigenfunction, spherical harmonics
$l(l+1)\hbar^2$ - eigenvalue

- $\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (Pauli spin matrices)
- $\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

These have the following properties:

$\hat{\sigma}_x^2 = \hat{\sigma}_y^2 = \hat{\sigma}_z^2 = I_{2\times 2}$ (2x2 identity matrix)

$[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_x$ etc.

$\hat{\sigma}_x \hat{\sigma}_y + \hat{\sigma}_y \hat{\sigma}_x = 0$
$\hat{\sigma}_y \hat{\sigma}_z + \hat{\sigma}_z \hat{\sigma}_y = 0$ (anti commute)
$\hat{\sigma}_x \hat{\sigma}_z + \hat{\sigma}_z \hat{\sigma}_x = 0$

- $|\Psi\rangle$ (ket)
- $\langle\Psi|$ (Bra) (complex conjugate)
- $\int \Psi^*(x)\, \hat{A}\, \Psi(x)\, dx = \langle\Psi | \hat{A}\Psi\rangle = \langle\Psi|\hat{A}|\Psi\rangle$

(Dirac notation)

- as $\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$ (eigenvalues of matrix)

$\underline{\underline{A}}$ - matrix we want eigenvalues from
λ - eigenvalues
$\underline{\underline{I}}$ - identity matrix

- $R = R_0 A^{1/3}$ (nuclear radius)

$R_0 = 1.2 \times 10^{-15}$

R - radius of nucleus

A - mass number

$\boxed{{}^{A}_{Z}He}$ A - mass number (4 for He)

Z - proton number (2 for He)

- $A = Z + N$ (mass number)

Z - proton number (no. of protons)

N - neutron number (no. of neutrons)

- $E_B = (Z(m_p + m_e) + Nm_n)c^2 - Mc^2$ (Binding energy)

E_B - binding energy

M - atomic mass

- $E_B = C_1 A - C_2 A^{2/3} - C_3 \dfrac{Z(Z-1)}{A^{1/3}} - C_4 \dfrac{(A-2Z)^2}{A} \pm C_5 A^{-4/3}$

C_5 is +ve if both Z, N even (liquid drop)

-ve if both Z, N odd

zero if Z and N are both odd and even

- $Z_{max} = \left(\dfrac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^{-1} = \alpha^{-1}$ (Theoretical proton limit)

α - fine structure constant ($\alpha \simeq \frac{1}{137}$)

- $-\hbar^2 \dfrac{\partial^2}{\partial t^2} \Psi(\vec{r}, t) = -\hbar^2 c^2 \nabla^2 \Psi(\vec{r}, t) + m^2 c^4 \Psi(\vec{r}, t)$ (Klein-Gordon equation)

This has plane wave ($\Psi(\vec{r}, t) = Ae^{i(\vec{p}\cdot\vec{r} - Et)/\hbar}$) solutions, provided, $E^2 = p^2c^2 + m^2c^4$.

- $(-i\hbar c\, \vec{\alpha} \cdot \nabla + \beta mc^2)\Psi(\vec{r}, t) = i\hbar \dfrac{\partial \Psi(\vec{r}, t)}{\partial t}$ (Dirac equation)

$\vec{\alpha}$ - vector made up of matrices

β - matrix

$\Psi(\vec{r}, t)$ - 4-component spinor

Mathematical Methods for Physicists

- $$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$ (partial derivatives)

  for well behaved functions, f.

- $$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$ (total derivative)

- $$\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}$$ (condition for exact differential)

  $$A = \frac{\partial f}{\partial x} \; ; \; B = \frac{\partial f}{\partial y}$$

- $$\frac{df}{ds} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds}$$ (chain rule)

  f is a function of x and y, $f(x,y)$.

- $$\nabla \phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$ (Grad vector)

  ϕ - scalar field, $\phi(x,y,z)$.

- $\nabla \phi \cdot \hat{u}$ (directional derivative)

  $\hat{u}$ - unit vector of $\vec{u}$

- $$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$ (Laplacian) (Cartesian)

  ϕ - Scalar field

- $$d(f + \lambda g) = \left(\frac{\partial f}{\partial x} + \lambda \frac{\partial g}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y} + \lambda \frac{\partial g}{\partial y}\right) dy = 0$$

  λ - Lagrange multiplier (Lagrange multiplier)

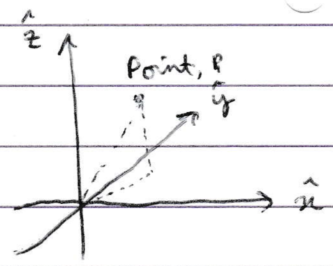
  f and g are functions of x and y.

- $\nabla f + \lambda \nabla g = 0$ (Lagrange multiplier using vectors)

- $$|J| = \frac{\partial(x,y)}{\partial(u,v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$ (Jacobian Matrix)

  u, v are the coordinates we wish to transform to.
  Multiply $|J|$ by $du\,dv$ to find area element.

Coordinate Systems



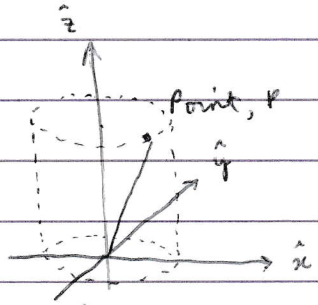
Cartesian: x, y, z $(-\infty < r < \infty)$

We have 3 orthogonal planes.

Area element: $dA = dx\,dy$ $= dx\,dz$ $= dy\,dz$ } depends on plane

Volume element: $dV = dx\,dy\,dz$

Basis vectors: $\hat{i}, \hat{j}, \hat{k}$



Cylindrical coordinates:

(radial) $R = \sqrt{x^2 + y^2}$ $(0 \leq R < \infty)$

(azimuth) $\phi = \arctan\left(y/x\right)$ $(0 \leq \phi < 2\pi)$

(axial) $z = z$ $(-\infty < z < \infty)$

This is when we are constrained to a cylinder

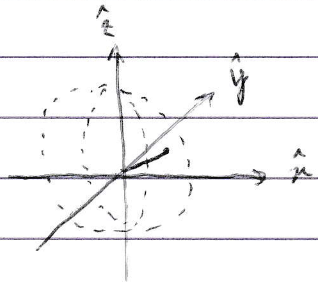
Area element: $dA = R\,d\phi\,dz$

Volume element: $dV = R\,dR\,d\phi\,dz$

Basis vectors: $\hat{R} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$

$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$

$\hat{z} = \hat{k}$



Spherical polar coordinates:

(radial) $r = \sqrt{x^2 + y^2 + z^2}$ $(0 \leq r < \infty)$

(polar) $\theta = \arccos\left(\frac{z}{r}\right)$ $(0 \leq \theta \leq \pi)$

(azimuthal) $\phi = \arctan\left(\frac{y}{x}\right)$ $(0 \leq \phi < 2\pi)$

This is when we are constrained to a sphere.

Area element: $dA = r^2 \sin\theta\,d\phi\,d\theta$

Volume element: $dV = r^2 \sin\theta\,d\phi\,d\theta\,dr$

Basis vectors: $\hat{r} = \cos\phi\sin\theta\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$

$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j} + 0\,\hat{k}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$

- $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ (Cartesian → cylindrical coords)
- $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ (Cartesian → spherical coords)
- $\bar{x} = \frac{1}{A}\iint x \, dA$ (centroid, geometrical centre.)

  A - total surface area
  $\bar{x}$ - x-coord of centroid.
- $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$ (arc length)

  a and b are the starting and finishing x-coords of the given arc.
- $A = \int_{s_1}^{s_2} 2\pi y \, ds$ (Surface area of revolution)

  integrating over the length of the curve, s.
- $A = 2\pi \bar{y} S$ (Pappus' 2nd theorem)

  S - ~~surface~~ arc length
  A - surface area of revolution
  $\bar{y}$ - y-coord of centroid of arc
- $V = \pi \int_{x_1}^{x_2} f^2(x) \, dx$ (Volume of revolution)
- $V = 2\pi A \bar{y}$ (Pappus' 1st theorem.

  V - volume of revolution.
  $\bar{y}$ - y-coord of centroid of triangular plane.

For a vector field, $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
And a scalar field, $\phi = \phi(x,y,z)$.

- Grad: $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

Returns the vector gradient of the scalar field

- Divergence: $\text{div}\,\vec{a} = \nabla\cdot\vec{a} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

This returns the scalar divergence of a vector field, ie. the net flux through a surface / volume.

- Curl: $\text{curl}\,\vec{a} = \nabla\times\vec{a} = \det\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ a_x & a_y & a_z\end{bmatrix}$

$$= \left[\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right]\hat{i} - \left[\frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z}\right]\hat{j} + \left[\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right]\hat{k}$$

Returns the vector curl of a vector field. This is interpreted as the tendency of the field to circulate.

- Laplacian: $\nabla^2\phi$ or $\nabla^2\vec{a}$

for scalar field: $\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$
$= \nabla\cdot(\nabla\phi)$

for vector field: $\nabla^2\vec{a} = (\nabla^2 a_x)\hat{i} + (\nabla^2 a_y)\hat{j} + (\nabla^2 a_z)\hat{k}$

where: $\nabla^2 a_x = \frac{\partial^2 a_x}{\partial x^2} + \frac{\partial^2 a_x}{\partial y^2} + \frac{\partial^2 a_x}{\partial z^2}$

$\nabla^2 a_y = \frac{\partial^2 a_y}{\partial x^2} + \frac{\partial^2 a_y}{\partial y^2} + \frac{\partial^2 a_y}{\partial z^2}$

$\nabla^2 a_z = \frac{\partial^2 a_z}{\partial x^2} + \frac{\partial^2 a_z}{\partial y^2} + \frac{\partial^2 a_z}{\partial z^2}$

- $\nabla \times (\nabla \phi) = 0$ (Curl of grad) } always true!
- $\nabla \cdot (\nabla \times \vec{a}) = 0$ (Div of curl)
- $$\nabla(\nabla \cdot \vec{a}) = \left[\frac{\partial^2 a_x}{\partial x^2} + \frac{\partial^2 a_y}{\partial x \partial y} + \frac{\partial^2 a_z}{\partial x \partial z}\right]\hat{i} + \left[\frac{\partial^2 a_x}{\partial y \partial x} + \frac{\partial^2 a_y}{\partial y^2} + \frac{\partial^2 a_z}{\partial y \partial z}\right]\hat{j} + \left[\frac{\partial^2 a_x}{\partial z \partial x} + \frac{\partial^2 a_y}{\partial z \partial y} + \frac{\partial^2 a_z}{\partial z^2}\right]\hat{k}$$ (Grad of a div)
- $\nabla \times (\nabla \times \vec{a}) = \nabla(\nabla \cdot \vec{a}) - \nabla^2 \vec{a}$ (curl of curl)
- $\nabla \cdot (\phi \vec{a}) = \vec{a} \cdot \nabla \phi + \phi (\nabla \cdot \vec{a})$
- $\nabla(\phi \phi') = \phi \nabla \phi' + \phi' \nabla \phi$ (product rule)
- $\frac{d}{dt}(\vec{a} \cdot \vec{b}) = \vec{a} \cdot \frac{d\vec{b}}{dt} + \vec{b} \cdot \frac{d\vec{a}}{dt}$ (time derivatives)
- $$\oint_c \vec{F} \cdot d\vec{r} = \oint_c (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

  usually set $P = -y$ and $Q = x$ (Green's theorem in the plane)
- $d\vec{s} = ds\,\hat{n}$ (vector surface element)

  $\hat{n}$ - normal vector to the surface

  Same applies for volume.
- $\oint_c \vec{F} \cdot d\vec{r} = \iint (\nabla \times \vec{F}) \cdot d\vec{s}$ (Stokes theorem)
- $\oiint_s \vec{F} \cdot d\vec{s} = \iiint_v (\nabla \cdot \vec{F})\, dV$ (Divergence theorem)
- $\iint \phi\, d\vec{s} = \iiint (\nabla \phi)\, d\vec{V}$ (Divergence theorem for scalar field)
- $\int_0^L c(x,t)\, dx = N$ (Concentration of particles in 1D)

N - number of particles

$c(x,t)$ - concentration in 1D

L - length of medium

- $j(x) = -D\dfrac{\partial c}{\partial x}$ (Fick's law)

D - diffusivity of medium
$j(x)$ - particle flux

- $\dfrac{\partial c}{\partial t} = D\dfrac{\partial^2 c}{\partial x}$ (Diffusion equation)

$c = c(x,t)$ - concentration
t - time
(This can be recast to give the heat transport equation)

- $\dfrac{\partial c}{\partial t} = \dfrac{\partial}{\partial x}\left(D\dfrac{\partial c}{\partial x}\right) + S(x,t)$ (More general diffusion equation)

$S(x,t)$ - source / sink of particles

- $\dfrac{\partial T(x,t)}{\partial t} = K\dfrac{\partial^2 T(x,t)}{\partial x^2}$ (Heat equation)

K - thermal diffusivity of medium
$T(x,t)$ - temperature

- $\dfrac{\partial^2 u}{\partial t^2} = c^2\dfrac{\partial^2 u}{\partial x^2}$ (Wave equation)

c - wavespeed
u - displacement / waveform

- $c = \sqrt{\dfrac{T}{\rho}}$ (Wavespeed of mechanical wave)

T - tension
ρ - mass per unit length

- $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\dfrac{n\pi x}{L}\right) + b_n \sin\left(\dfrac{n\pi x}{L}\right)$ (Fourier series)

- $\dfrac{a_0}{2} = \langle f(x)\rangle = \dfrac{1}{2L}\displaystyle\int_{-L}^{L} f(x)\,dx$
- $a_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\cos\left(\dfrac{n\pi x}{L}\right)dx$
- $b_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\sin\left(\dfrac{n\pi x}{L}\right)dx$

(Fourier coefficients for $-L \le x \le L$)

- $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\frac{n\pi x}{L}}$ (Complex Fourier series)
- $c_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-\frac{in\pi x}{L}} dx$ (Complex Fourier coefficient)
- $f(-x) = f(x)$ (Even functions)
- $f(-x) = -f(x)$ (Odd functions)

(sin (x) is odd, cos (x) is even.)

- $\int_{-L}^{L} \cos\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = 0$
- $\int_{-L}^{L} \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right) dx = L\delta_{mn}$
- $\int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = L\delta_{mn}$

(Orthogonality relations)

- $\delta_{mn} = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$ (Kronecker delta)
- $\delta(x) = \begin{cases} 0 & |x| > \frac{\omega}{2} \\ \frac{1}{\omega} & |x| \leq \frac{\omega}{2} \end{cases}$ (as $\omega \to 0$) (Dirac Delta function)
- $\delta(x - x_0) = \begin{cases} 0 & \text{for } x \neq x_0 \\ \infty & \text{for } x = x_0 \end{cases}$
- $\int \delta(x - x_0)\, dx = \begin{cases} 1 & \text{if range of integration contains } x_0 \\ 0 & \text{otherwise} \end{cases}$
- $\int f(x)\,\delta(x - x_0)\, dx = \begin{cases} f(x_0) & \text{if range contains } x_0 \\ 0 & \text{otherwise} \end{cases}$
- $u(0,t) = u(L,t) = 0 \ \forall t$ 'clamped string'
- $\left.\frac{\partial c}{\partial x}\right|_{x=0} = \left.\frac{\partial c}{\partial x}\right|_{x=L} = 0 \ \forall t$ 'constant flux'
- $u(x,0) = f(x)$ 'initial shape'
- $\left.\frac{\partial u}{\partial t}\right|_{t=0} = V_0\,\delta(x - x_0)$ 'impulse'

(Boundary conditions for PDEs)

$$\frac{\partial^2 u(\vec{r},t)}{\partial t^2} = c^2 \nabla^2 u(\vec{r},t)$$ (3D wave equation)

$u(\vec{r},t) = C' e^{i(\vec{k}\cdot\vec{r} \pm \omega t)}$ (Solution to 3D wave eqn) (Plane waves)

C' - amplitude

$\vec{k}$ - wave vector

$\vec{r}$ - position vector

ω - angular velocity

$\omega^2 = |\vec{k}|^2 c^2$ (Dispersion relation)

c - wavespeed

$\nabla^2 \phi = \text{constant}$ (Poisson's equation)

$\nabla^2 \phi = 0$ (Laplace's equation)

∇^2 - Laplacian operator ($\nabla^2 = \nabla \cdot \nabla$)

$$F(f(x)) = \tilde{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$ (Fourier transform)

$$F^{-1}(\tilde{f}(k)) = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(k) e^{ikx} dk$$ (Inverse transform)

Can find lots of standard Fourier transforms either online / in PX275 notes.

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$ (Gaussian integral)

α - some constant

$$\int_{-\infty}^{\infty} |u(x)|^2 dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\tilde{u}(k)|^2 dk$$ (Parseval's theorem)

$$\int_{-\infty}^{\infty} f(x) g^*(x) dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(k) \tilde{g}^*(k) dk$$ (Generalised Parseval's theorem)

$$f * g(x) = \int_{-\infty}^{\infty} f(k) g(x-k) dk$$ (Convolution theorem)

k is some dummy variable.

$$F(f * g(x)) = \tilde{f}(k)\tilde{g}(k)$$ (F.T. of convolution)

$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$ (Heaviside step function)

- $$\tilde{f}(k) = \int_{\mathbb{R}^N} e^{-i(\vec{k}\cdot\vec{r})} f(\vec{r})\, d^N\vec{r}$$
- $$f(\vec{r}) = \left(\frac{1}{2\pi}\right)^N \int_{\mathbb{R}^N} e^{i(\vec{k}\cdot\vec{r})} \tilde{f}(\vec{k})\, d^N\vec{k}$$

(Generalised Fourier transform in N dimension.)

- $$f * g(\vec{x}) = \int_{\mathbb{R}^N} f(\vec{z})\, g(\vec{x}-\vec{z})\, d^N\vec{z}$$ (Generalised convolution in N dimensions)

$\mathbb{R}^N$ signifies that N integrals need to be evaluated over all space.

- $$\mathcal{F}\left(\frac{\partial^n f}{\partial x^n}\right) = (ik)^n \tilde{f}(k)$$ (F.T. of differential equation)
- $n\lambda$ = constructive; $(n+\frac{1}{2})\lambda$ = destructive (Path length and interference)
- $u \propto \frac{1}{r}$ (Amplitude of 3D point source)
- $I(r) = |u|^2 \propto \frac{1}{4\pi r^2}$ (Intensity of 3D point source)
- $$u(r,t) = \frac{A}{r} e^{ik(r-ct)}$$ (Wave eqn solution for point source)

A - amplitude
r - radial distance from source
k - wavenumber

- $$u(\vec{x},t) = \sum_{i=1}^{N} \frac{A^i}{|\vec{x}-\vec{y}^i|} e^{ik(|\vec{x}-\vec{y}^i|-ct)}$$ (Pinhole aperture sources)

$\vec{x}$ - vector on detector screen
$\vec{y}$ - vector on aperture screen

- $\frac{|\vec{y}|}{|\vec{x}|} \ll 1 \Rightarrow |\vec{x}| \gg |\vec{y}|$ (Far-field approximation)
- $|\vec{x}-\vec{y}| \simeq |\vec{x}| - \vec{k}\cdot\vec{y}$

$\vec{k}$ - wavevector, $(\vec{k} = k\hat{x})$

- $u(x) \propto \hat{a}(k)$
- $I(\vec{x}) = |u|^2 \propto |\hat{a}(\vec{k})|^2$ (Aperture functions)

Electromagnetic theory and optics

- $$\oiint_S \vec{W} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{W})\, dv$$ (Divergence theorem)

$\vec{W}$ - vector field
$d\vec{s}$ - vector surface element
dv - scalar volume element
∇ - grad operator

- $$\oint_C \vec{W} \cdot d\vec{l} = \iint_S (\nabla \times \vec{W}) \cdot d\vec{s}$$ (Stokes's theorem)

$d\vec{l}$ - vector line element

(Some vector identities)

- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
- $\nabla \times (\nabla \times \vec{W}) = \nabla(\nabla \cdot \vec{W}) - \nabla^2 \vec{W}$
- $\nabla \cdot (f\vec{W}) = (\nabla f) \cdot \vec{W} + f(\nabla \cdot \vec{W})$
- $\nabla \times (f\vec{W}) = (\nabla f) \times \vec{W} + f(\nabla \times \vec{W})$
- $\nabla \cdot (\vec{V} \times \vec{W}) = \vec{W} \cdot (\nabla \times \vec{V}) - \vec{V} \cdot (\nabla \times \vec{W})$

$\vec{a}, \vec{b}, \vec{c}, \vec{W}, \vec{V}$ are all vector fields
f is a scalar field

- $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ (Lorentz force)

$\vec{F}$ - force vector
q - charge on particle
$\vec{E}$ - Electric field
$\vec{v}$ - velocity of particle
$\vec{B}$ - magnetic flux density

- $$\oiint_S \vec{E} \cdot d\vec{s} = \frac{Q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \iiint_V \rho \, dv$$ (Gauss' law, integral form)

$Q_{encl.}$ - enclosed charge by Gaussian surface, S.
ε_0 - permittivity of free space
ρ - charge density

- $$\oiint_S \vec{B} \cdot d\vec{s} = 0$$ (Solenoidal condition)

Implies there are no magnetic monopoles

PX265

- $\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S} = -\frac{d\Phi_B}{dt}$ (Faraday-Lenz law)

$\mathcal{E}$ - emf induced
Φ_B - magnetic flux
Contour C is the boundary of surface S.

- $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl.} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$ (Ampère's law, integral form)

μ_0 - permeability of free space
$I_{encl.}$ - current enclosed by Ampèrean loop
$\vec{J}$ - current density vector

- $\vec{J} = \frac{I}{A} = \rho \vec{v}$ (Current density)

A - cross-sectional area
$\vec{v}$ - velocity of electrons

- $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$ (Continuity equation for electric charge)

(Maxwell's Equations in free space):

- $\nabla \cdot \vec{E} = \rho/\varepsilon_0$ (Gauss' law, derivative form)
- $\nabla \cdot \vec{B} = 0$ (Solenoidal condition)
- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday-Lenz law)
- $\nabla \times \vec{B} = \mu_0\left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$ (Ampère's law)

(EM wave equations):

- $\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$
- $\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$

- $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8 \, ms^{-1}$ (Speed of light)
- $\frac{|\vec{E}|}{|\vec{B}|} = \frac{\omega}{k} = c$ (EM waves)

ω - angular velocity

$\partial_t \to -i\omega$ (Working with plane waves)

$\nabla \to i\vec{k}$

∂_t - time derivative

$\vec{k}$ - wavevector

$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ (Poynting vector)

$u = \frac{B^2}{2\mu_0} + \frac{\varepsilon_0 E^2}{2}$ (Energy density)

$\sigma_p = \vec{P} \cdot \hat{n}$ (Surface charge density)

σ_p - surface charge density

$\vec{P}$ - Polarisation

$\hat{n}$ - normal unit vector

$\rho_p = -\nabla \cdot \vec{P}$ (Volume charge density)

ρ_p - volume charge density

$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$ (Polarisation current density)

$\vec{J}_p$ - Polarisation current density vector

$\vec{P} = \varepsilon_0 \chi \vec{E}$ (isotropic electric field polarisation)

χ - susceptibility / polarisability

$\varepsilon_r = 1 + \chi$ (relative permittivity)

$\vec{j}_M = \vec{M} \times \hat{n}$ (Surface current density)

$\vec{M}$ - magnetisation vector

$\vec{J}_M = \nabla \times \vec{M}$ (volume magnetisation current)

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (displacement)

$\vec{P}$ - polarisation vector

$\vec{D}$ - displacement vector

$\nabla \cdot \vec{D} = \rho_f$ (Modified Gauss' law)

ρ_f - free charge density

$\vec{D} = \varepsilon_0 (1 + \chi) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ (Displacement in matter)

ε_r - relative permittivity

- $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ (Magnetic field strength)

$\vec{H}$ - magnetic field strength
$\vec{B}$ - magnetic flux density
$\vec{M}$ - magnetisation

- $\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$ (Ampère's law, alternate form)

$\vec{J}_f$ - free current density

- (Note: solenoidal condition and Faraday-Lenz remains the same in matter)

- $\vec{E}_{2,\parallel} = \vec{E}_{1,\parallel}$ (Boundary conditions on EM fields)

alternatively:

$\hat{n} \times (\vec{E}_2 - \vec{E}_1) = 0$

$\hat{n}$ - unit vector pointing normal to fields.

- $D_{1,\perp} - D_{2,\perp} = \sigma_f$ (Boundary condition)

σ_f - free surface charge density

alternatively:

$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \sigma_f$

- $\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$ (More boundaries)
- $\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_f$
- $\nabla^2 \vec{E} = \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial^2 \vec{E}}{\partial t^2}$ (Wave equation in dielectric)

μ_r - relative permeability
ε_r - relative permittivity

- $v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$ (Wave speed in dielectric)
- $n = \sqrt{\varepsilon_r \mu_r} = \frac{c}{v}$ (Refractive index)

$n \geq 1$

- $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73\,\Omega$ (Impedance of free space)

- $\vec{k}\cdot\vec{D} = 0$
- $\vec{k}\cdot\vec{B} = 0$
- $\vec{k}\times\vec{E} = \omega\vec{B}$
- $\vec{k}\times\vec{H} = -\omega\vec{D}$

(linear isotropic relations)

$\vec{k}$ - wavevector
$\vec{D}$ - displacement
$\vec{H}$ - magnetic field strength

- $\vec{S} = \vec{E}\times\vec{H}$ (Poynting vector)

$\vec{S}$ - Poynting vector

- $u = \frac{1}{2}(\vec{B}\cdot\vec{H} + \vec{E}\cdot\vec{D})$ (energy density)

u - energy density

- $\theta_i = \theta_r$
- $n_1 \sin\theta_i = n_2 \sin\theta_t$

(Snell's law)

θ_i - angle of incidence
θ_r - angle of reflection
θ_t - angle of transmitted wave.
n_1 - refractive index of 1st medium
n_2 - refractive index of 2nd index

- $r_s = \cancel{2n_1\cos}\ \frac{E_r}{E_i} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$
- $t_s = \frac{E_t}{E_i} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}$
- $r_p = \frac{E_r}{E_i} = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$
- $t_p = \frac{E_t}{E_i} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}$

(Fresnel relations)

r_s and t_s are reflection and transmission coefficients for S-polarisation. r_p and t_p are reflection and transmission coefficients for P-polarisation.

$\tan \theta_B = \frac{n_2}{n_1}$ (Brewster angle)

θ_B - Brewster angle, angle of incidence

$\sin \theta_{ic} = \frac{n_2}{n_1}$ (Critical angle for TIR)

θ_{ic} - critical angle of incidence.

$\delta = \sqrt{\frac{2}{\mu g \omega}}$ (Skin effect)

δ - skin depth

g - conductivity

μ - $\mu_0 \mu_r$