PX144: Intro to Astronomy

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Contents:

Aims:

To introduce the constituent objects of the Universe and the physics which allows us to estimate their distances, sizes, masses and natures. The module will show how our knowledge of the Universe beyond Earth relies entirely upon the application and extrapolation of physics developed in the laboratory. This will develop an appreciation of the wide range of applicability of physical principles, while touching upon areas under active development. The module will help the development of problem solving skills.

Disclaimer:

This revision guide has been put together using the sweat, blood and tears of the Maths and Physics Society members. **It is intended as a revision tool and as such does not contain the entirety of the module - only the key elements. It was not complied, nor ratified by the University and most certainly not collated by any past, present or future lecturer of the referred course. As such it may be incomplete and may contain slight inconsistencies and inaccuracies; although the utmost care has gone into ensuring that these are minimal.** Should you find any of the above mentioned please accept our apologies and find the time to drop us an email at the following address so we can change it in future editions: su442@sunion.warwick.ac.uk

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Based off 2008 lecture notes and online slides 2011.

1. Celestial Sphere and Time-Keeping

1.1 Historical Perspective

Astronomy is one of the oldest sciences in the world and was initially driven by religious/time keeping motivations. Astrological nomenclature is still heavily influenced by tradition – so watch out for unintuitive units/scales!

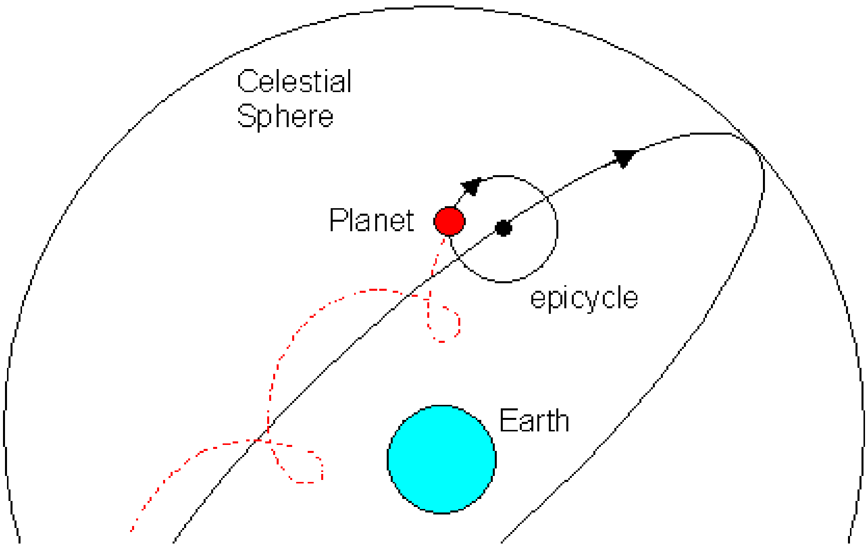
Plato ~400BC believed in a geocentric universe where the motions of all the heavenly bodies followed combinations of simple circular motions and challenged his Athenian colleagues to prove it.

Aristotle ~350BC extended this view to 55 concentric (on the Earth) crystalline spheres to which all of the celestial bodies were attached. By adjusting the velocities of these concentric spheres, many features of planetary motion could be explained. However, the troubling observations of varying planetary brightness and retrograde motion could not be accommodated: the spheres moved with constant angular velocity, and the objects attached to them were always the same distance from the earth because they moved on spheres with the earth at the centre.

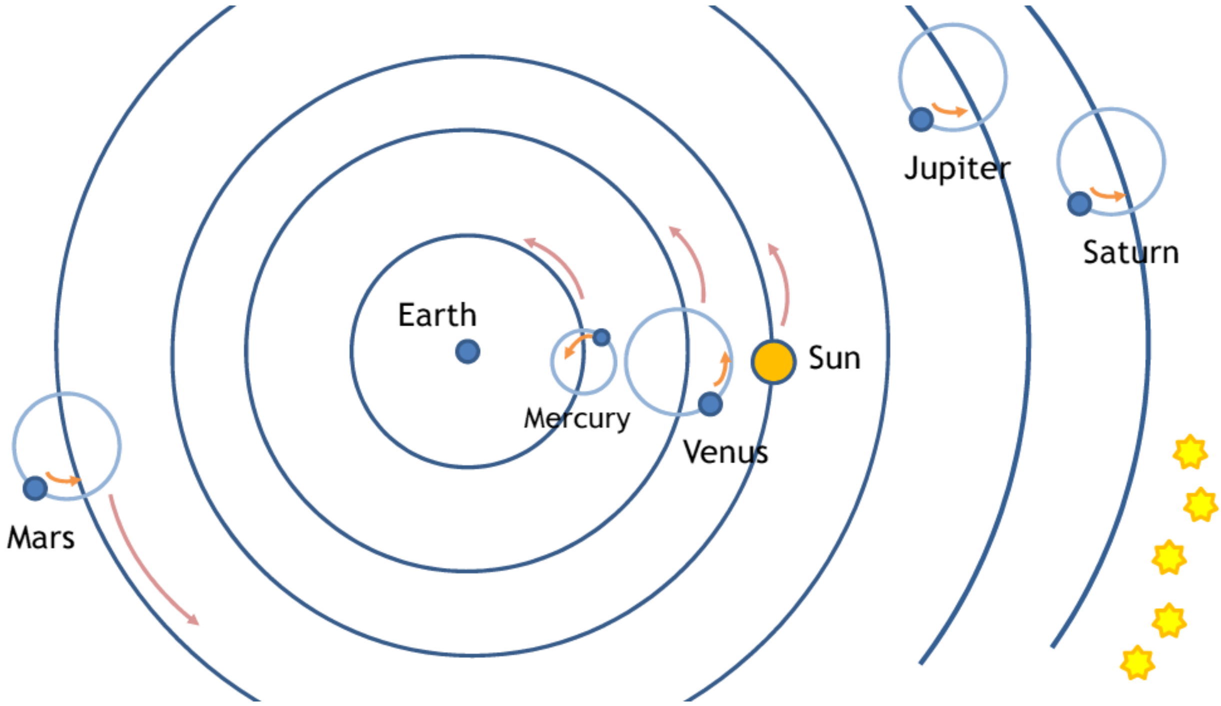
Retrograde Motion.

Aristarchus ~250BC presented the first heliocentric model of the universe; however his theory was not widely adopted until 1800 years later.

Hipparchus ~130BC too believed in a geocentric universe, proposing epicycles to better model the motion of celestial bodies.



Ptolemy ~150AD (200 years later!) published the Almagest, an Ancient Greek mathematical and astronomical treatise on the complex motions of the stars and planetary paths based on a geocentric universe and epicycles. This model was accepted as dogma for 1200 years.



Copernicus 1543AD (Copernican Revolution!) using a heliocentric model, demonstrated that the observed motions of celestial objects can be explained without putting Earth at rest in the center of the universe. Offering an alternative to the Ptolemy model (not immediately accepted).

Brahe 1573AD was a great observer and noted huge discrepancies in the Ptolemy model.

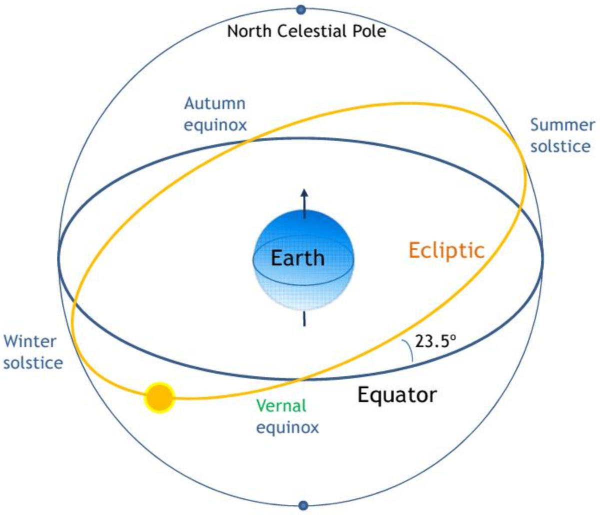
Kepler ~1610AD was a Copernican proponent, with his works providing the basis for Isaac Newton's theory of Universal Gravitation.

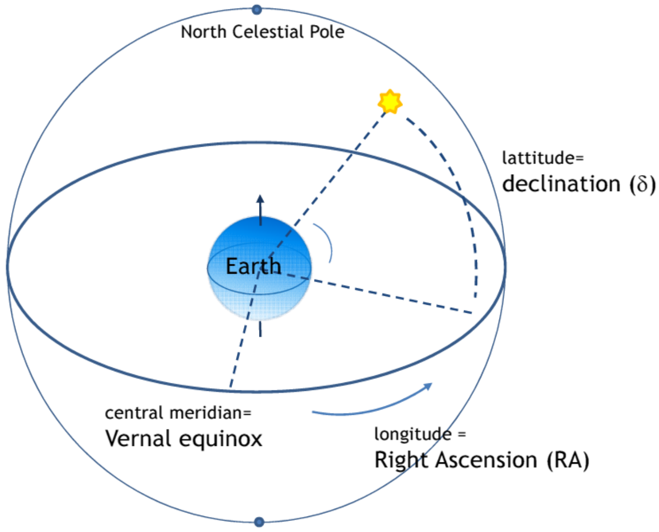
Galileo 1610AD was an empirical scientist and supported the heliocentric model to much controversy. His observations with a telescope (not least observing the orbit Jupiter's moons – not geocentric) supported the Copernican model.

Newton 1687AD in Principia Mathematica provided a consistent physical explanation which showed that the planets are kept in their orbits by the familiar force of gravity. Newton was able to derive Kepler's laws as good approximations and to get yet more accurate predictions by taking account of the gravitational interaction between the planets. Newton arguably completed the Copernican Revolution although the Ptolemaic model was not fully abandoned until the 18th century.

1.2 Celestial Sphere

We live on a rotating reference frame, and depending on your latitude on the Earth you have a different horizon. Hence the local reference frame is not very good as it depends on location and time. Hence we always consider the Celestial Sphere.





1.3 Time Keeping

The Julian Calendar 45BC introduced the 365 day year with a 366 day leap year every 4 years, and included the modern 30/31 day months.

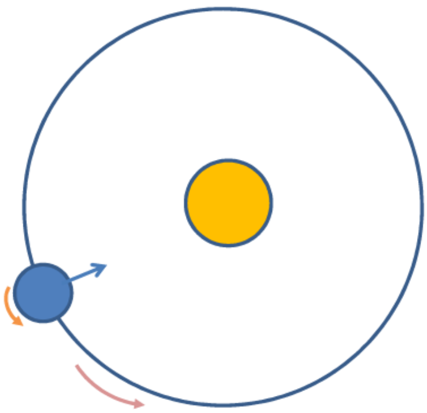
However this calendar was too long, slipping 0.78days/century. The Gregorian Calendar (1582AD – current) stipulates that years divisible by 100 but not 400 are not leap years. (So 2100 will be not be a leap year).

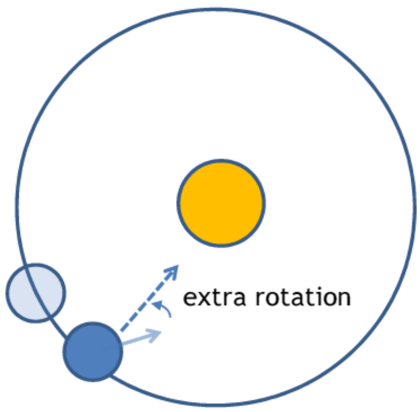
The current length of a year (equinox to equinox) is 365.2422 SI days. The length of a day was defined as one solar day in 1820, 24:00:00 – although the true day is slowly lengthening due to tidal effects.

Time is now measured using very stable atomic clocks. Leap seconds (±1) are sometimes made to keep the Coordinated Universal Time close to Mean Solar Time.

We originally defined a day as the time for the Sun to travel from South to South on the horizon. This is one solar day or one synodic day. Solar = synodic = 24:00:00.

However the real time it takes the Earth to rotate is 23:56:04, one Sidereal day.





The difference comes from our moving reference point as we orbit the Sun. It takes 4 minutes longer for the reference point to reach the Sun than it takes the Earth to truly rotate.

$$\frac{1}{P_{Synodic}} = \frac{1}{P_{Sidereal}} - \frac{1}{P_{Year}}$$

2. Angle, Parallax & Distances

2.1 Angles

Thanks to the Babylonians we express angles in the sexagesimal (base-60) system:

360 degrees = 2π radians
15 degrees = 1 hour (divides a circle into 24 hours)
1 degree = 60 arc minutes
1 arc minute = 60 arc seconds

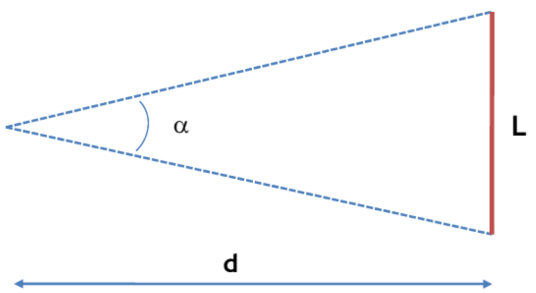
Notation: 16d 28m 40s or: 16^0 28' 40" or: 16:28:40

Careful of the hour notation: 30h 31m 32s = 2d 31m 32s (easily missed).
Right ascension is often expressed in hour angle.

Due to the large distances involved in astronomy, angles are often very small. Using Taylor series and neglecting higher order terms we often use the **small angle approximations:**

$$\sin(x) \cong \tan(x) \cong x \qquad and \qquad \cos(x) \cong 1 - x^2 \quad (x \; in \; radians)$$

For non-point like objects we can refer to the **angular size of the object:**

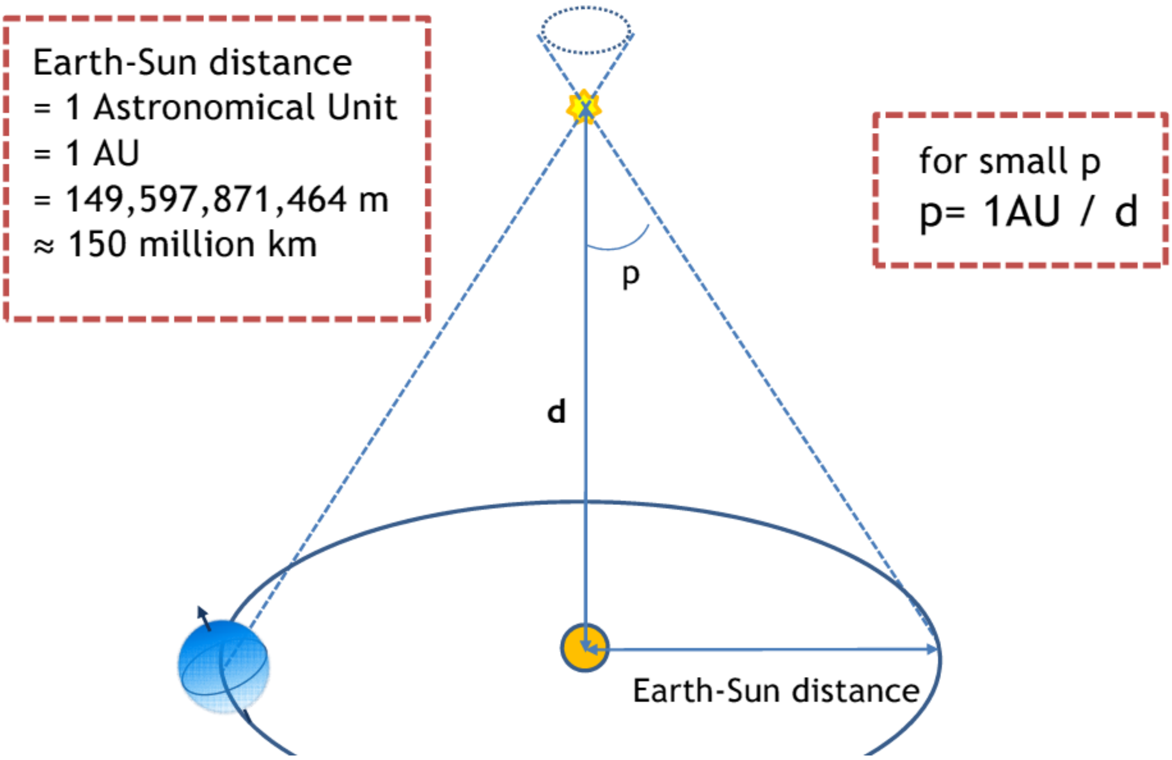


Here L is the length of the object, d the distance to the object and α the angular size. Usually d>>L so using small angle formula:

$$\tan(\alpha/2) = L/2d \qquad \therefore \qquad \boldsymbol{\alpha = L/d}$$

2.2 Parallax and Distance

The ancients had very accurate star charts giving the positions of celestial bodies. However problems arise when trying to measure distance to objects when all we have is angle – even today. One other method of finding distance is trigonometric **parallax** where we exploit the orbit of the Earth.



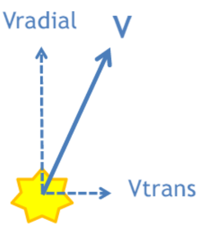
We know the Earth-Sun distance (1AU) and can measure the parallax angle over a year. Hence we can find distance using the small angle formula: $d(km) = 1AU(km)/p(rad)$

However this is a little inconvenient. So we define one parsec (pc) as the distance corresponding to a parallax angle of 1 arc second (1"): $\boldsymbol{d(pc) = 1/p''}$

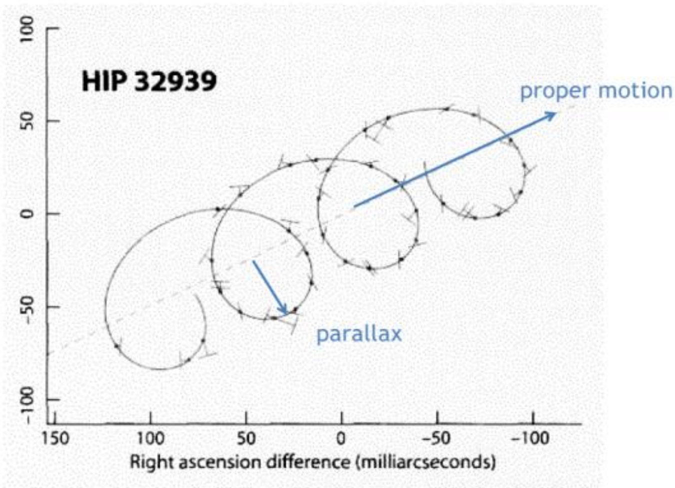
$$1pc \cong 3.09 \times 10^{13} km \cong 3.26\ light\ years$$

For example the closest star is Proxima Centauri at 0.772" (1.29pc). the first parallax measurement was made by Bessel in 1838 for 61 Cygni at 0.3" (modern value: 0.287").

Measuring typical parallax is not easy as stars have velocity in space too. So we often use more distant, stable reference to correct for proper motion. By measuring angle/position over long timescales, when the transverse component of the drift velocity (the **proper motion**) is found, the parallax can be measured.



Example Hipparchus satellite data:



3. Fluxes, Luminosities & Magnitudes

Note: <u>Really</u> learn these equations!

Brightness: is the received light power from an object. Brightness is not an intrinsic property of a star but is "apparent" and depends on your location.

Flux (f): is the received light power per unit area from an object [W/m^2].

Luminosity (L): is the intrinsic power at the source i.e. the total energy rate emitted [W].

Luminosity is found by integrating flux (usually over a sphere): $\boldsymbol{L = \oint f\, dA}$

Flux is luminosity over area (so drops off with distance): $\boldsymbol{f = L/4\pi d^2}$

There are two types of "magnitude" which is a measure of brightness. **Apparent magnitude (m)** is the brightness of a body as measured from Earth and is not intrinsic. However, **absolute magnitude (M)** is the intrinsic brightness of a body as if it was measured from 10pc away.

Hipparchus invented the logarithmic magnitude scale we still more or less use as our eyes have a logarithmic response. **<u>Brighter stars have a lower magnitude.</u>** Vega is defined to have zero apparent magnitude **Vega m=0**. This means stars brighter in the sky than Vega have negative apparent magnitude (Sun m=-26.74).

Pogson 1896 decided to formally fix the magnitude scale such that a change in 5 orders in magnitude equates to a factor of 100 in brightness. Therefore one magnitudes difference is a factor of $100^{1/5}$ = 2.51x ≈ 2.5 leading to the formula:

$$\boldsymbol{m_1 - m_2 = -2.5 log_{10}\left(\frac{f_1}{f_2}\right)}$$

In turn we can now substitute in our flux equation at 10pc to find absolute magnitudes (remember your log properties and that luminosity is intrinsic so cancels):

$$\boldsymbol{m - M = 5 log_{10}\left(\frac{d}{10pc}\right)}$$

4. Telescopes and Instruments

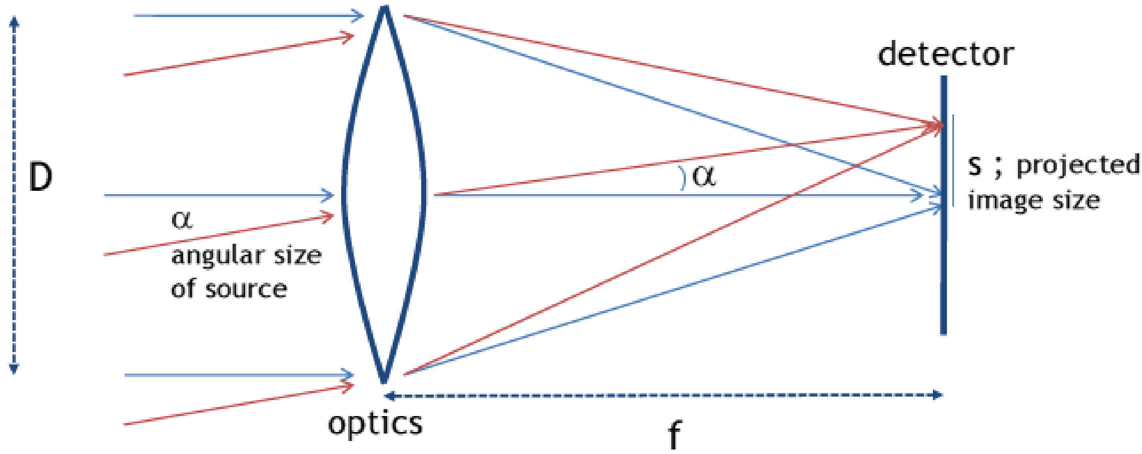
4.1 Light and the Eye

Visible light is a tiny window in the electromagnetic spectrum. Electromagnetic waves can have any wavelength but their speed in a vacuum is always c. $f = \frac{c}{\lambda}$ $E = hf$ etc

The human eye contains two types of photoreceptor cells in the retina: cones and rods. Cones are colour sensitive and come in three varieties: L (long), M (medium) and S (short), loosely corresponding to red/yellow, green and blue respectively. Rods do not perceive colour but are instead far more sensitive to light intensity (100x) – which is why we lose our colour vision as it gets dark.

4.2 Cameras

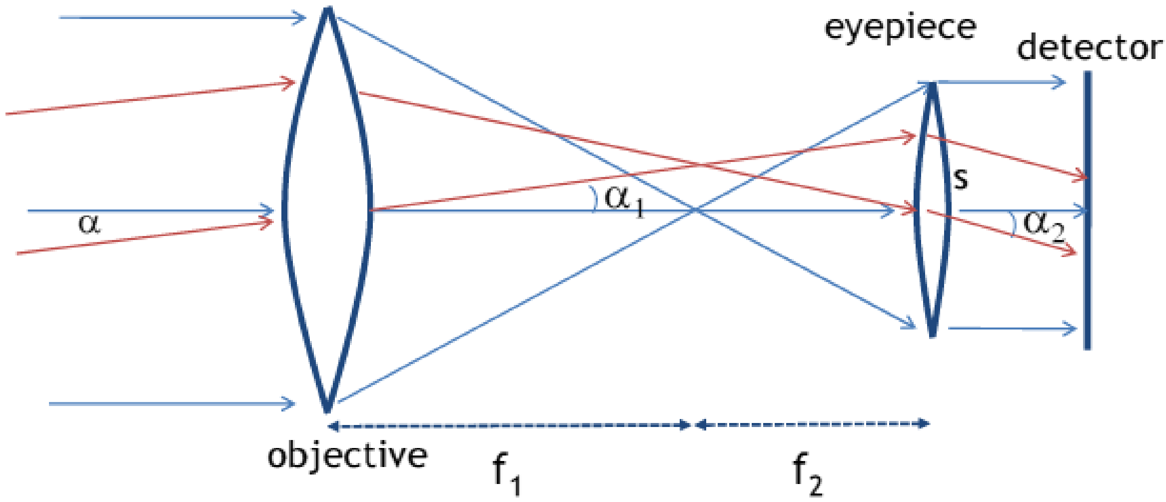
A camera has the following format:



D is the aperture diameter, f the focal length and s the projected image size.
Since we're dealing with small angles: $s = f\alpha$
The collecting area is $\pi(D/2)^2$ and the image size is $\pi(s/2)^2$ hence the brightness per unit area scales as: $(D/f)^2$
The "fastness" of a camera or the f-ratio is f/D - the smaller the better.

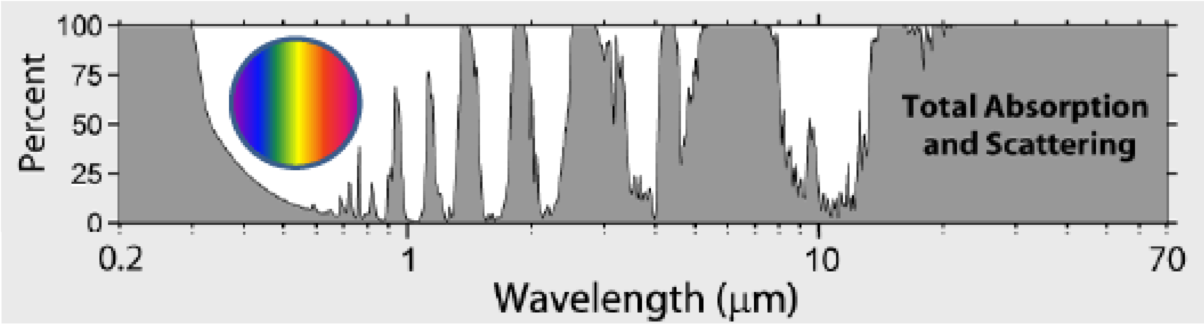
4.3 Telescopes

Add an eye piece to a camera with focal length f_2 and we get a telescope:



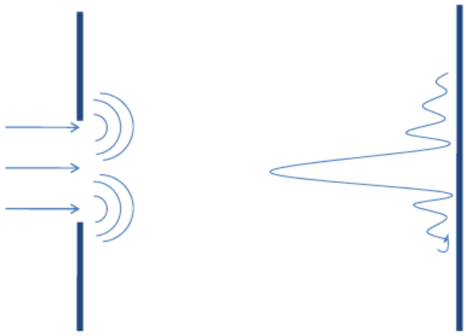
Using small triangles: $s = f_1\alpha_1 = f_2\alpha_2$
Hence the magnification factor is: $M = \frac{\alpha_2}{\alpha_1} = \frac{f_{objective}}{f_{eyepiece}}$
In reality, a variety of lenses and mirrors are used in different layouts in modern telescopes. The wavelengths you are trying to observe and the size of your aperture imposes limitations on the resolution you can achieve.

Atmospheric absorption means only a few windows of the electromagnetic spectrum are visible on Earth – and even then are limited by humidity/weather/turbulence etc.

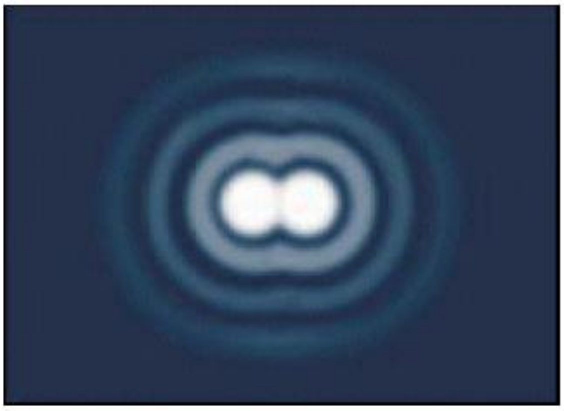


Hence we often use arrays of telescopes to combine images, in areas of low light pollution e.g. deserts. Or use space-based telescopes, which can see in e.g. infra-red, to achieve some amazing results.

Radio waves have long wavelengths, so radio astronomy requires very large apertures/reflector dishes.



However, even the largest telescopes are limited by diffraction based resolution (above – interference creates diffraction patterns). Point sources create Airy disc diffraction patterns. In order for them to be resolved they must be separated by an Airy disk ∴ satisfy: $\Delta\boldsymbol{\alpha} \geq \mathbf{1.22}\boldsymbol{\lambda}/\boldsymbol{D}$

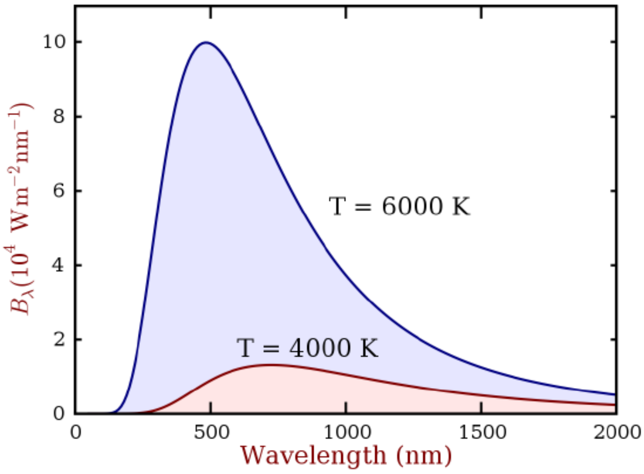


5. Colours, Temperatures and Blackbodies

5.1 Blackbodies

A black body is an idealized physical body that absorbs all incident electromagnetic radiation. Because of this perfect absorptivity at all wavelengths, a black body is also an ideal emitter of electromagnetic radiation, which it radiates incandescently in a characteristic, continuous spectrum that depends on the body's temperature. This spectrum is known as black-body radiation and is described by the Plank function: $I(\lambda) = B_\lambda(T) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}$

Don't need to remember the equation for this course, but do learn the distribution:



There are several important features of this distribution you need to know:

- The distribution varies smoothly with wavelength.
- There is a slowly dropping off tail to high wavelengths: **Raleigh-Jeans Tail**
- There is a sharp cut off at short wavelengths: **Wien cut-Off**
- $I(\lambda) = B_\lambda(T)$ increases at all wavelengths as T increases.
- The peak moves to shorter λ as T increases. Wien's Law. Hence hotter bodies appear bluer.

Implying these equations: (learn!)

- Wien's Law - BB emission peaks at: $\boldsymbol{\lambda(m)_{max} = 2.90 \times 10^3/T(K)}$
- Stefan-Boltzmann Law – surface flux is integrated power over wavelength, hence: $\boldsymbol{F = \int B_\lambda d\lambda = \sigma T^4}$ where $\sigma = 5.67 \times 10^{-8} Wm^{-2}K^{-4}$ is the Stefan-Boltzmann constant.
- Hence for a spherical source: $\boldsymbol{L = 4\pi R^2 F = 4\pi R^2 \sigma T^4}$

Finally, even if an object is not a true black body, it often resembles one so we define its effective temperature as: $F = \sigma {T_{eff}}^4$

5.2 Colour and Filters

Using Wien's law, if we can measure the spectrum of an object we can estimate its temperature. Since $I(\lambda)$ varies smoothly we can measure the light in a few specific intervals and compare the measured fluxes – filters.

Similar to how the eye has three cone type cells, three astronomical filters are often used: B blue, V visible and R red. (In the optical spectrum, U is for Ultra Violet etc). Defining magnitudes within each filter using references we can also define colour e.g. B-V.

$$m_B - m_{Bref} = -2.5log_{10}\left(\frac{f_B}{f_{Bref}}\right) \qquad m_V - m_{Vref} = -2.5log_{10}\left(\frac{f_v}{f_{Vref}}\right)$$

$$\boldsymbol{B - V \equiv m_B - m_V = -2.5log_{10}\left(\frac{f_B}{f_v}\right) + C}$$

Hotter objects are brighter in B, so $B < V$ and have smaller $B - V$ values, which is a measure of temperature.

6. Orbits and Masses

6.1 Kepler's Third Law

Newtonian Gravity: $F_G = {GMm}/{r^2}$ $\qquad G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

For $m \ll M$ at the surface radius R then acceleration due to gravity: $g = GM/R^2$

For one mass to orbit another where $m \ll M$, then gravitational attraction must balance centripetal acceleration in equilibrium. $F_C = mv^2/r$

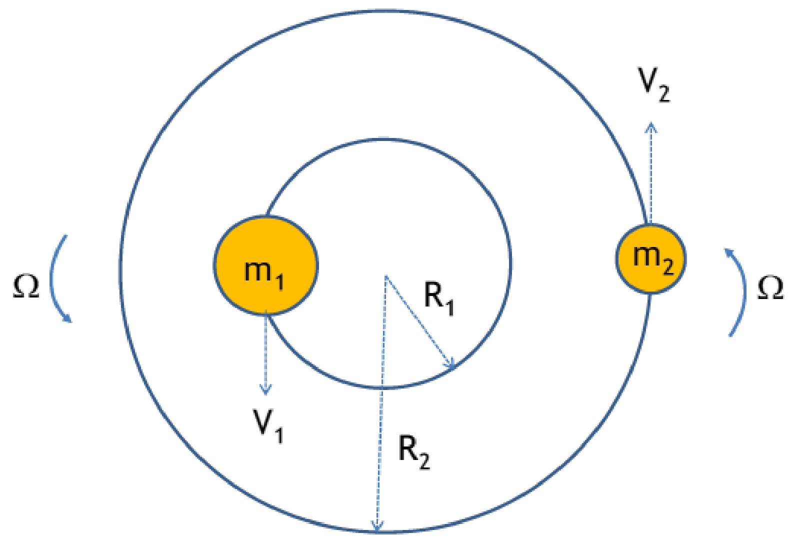
$v = \Omega r$ $\qquad \Omega = 2\pi/P$ $\qquad \Omega$ is angular velocity(s^{-1}) P is period of orbit (s)

$F_G = F_C = \frac{GMm}{r^2} = \frac{mv^2}{r} = m\Omega^2 r$ $\qquad$ Hence: $\qquad \Omega^2 = GM/r^3$

Which is Kepler's 3rd Law: $\qquad P^2 \propto R^3$

6.2 Binary Systems

In the general case two bodies m_1 and m_2 can be considered to make circular orbits around their centre of mass.



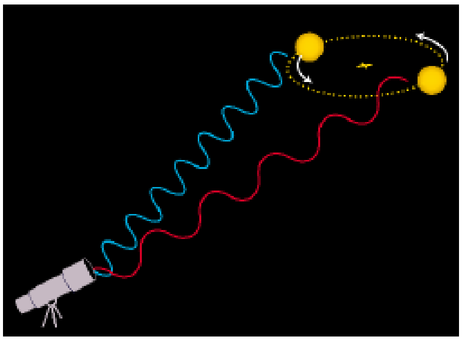
Centre of mass is origin thus: $m_1 r_1 = m_2 r_2$ Define: $R = r_1 + r_2$ and balance the forces:

$$\Omega^2 m_2 r_2 = \frac{G m_1 m_2}{R^2} = \Omega^2 m_1 r_1$$

Leading to the general Kepler's third law:

$$\Omega^2 = \frac{G(m_1 + m_2)}{R^3} = 4\pi^2/P^2$$

Binary star systems are very common in the galaxy and their orbits allow many properties to be directly determined. Often binaries eclipse which means we can find their period from the brightness oscillations. If we cannot also observe their orbital radii visually then we can calculate their orbital radial velocity using the Doppler Effect of their emission (giving us all the information we need).

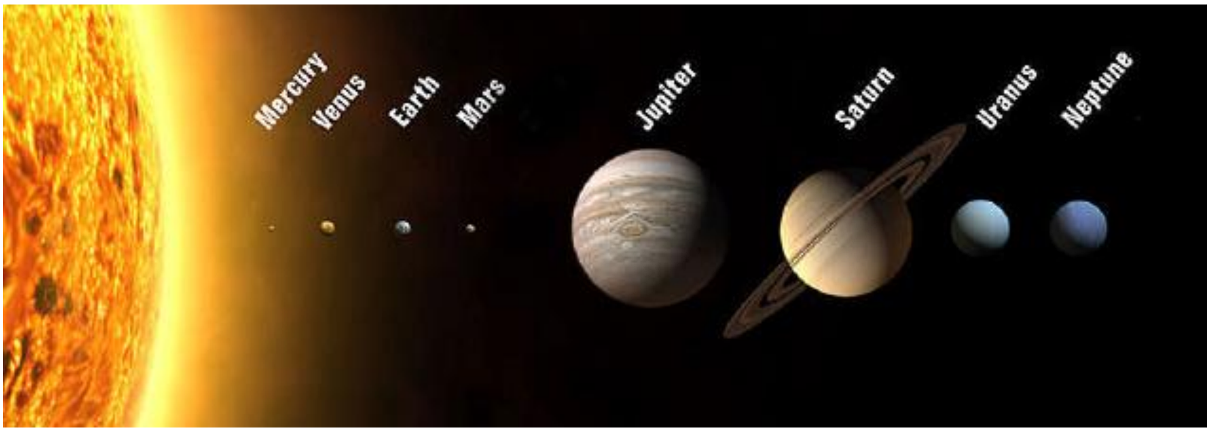


The Doppler Effect shifts characteristic emission lines meaning we can find radial velocity knowing the original and received wavelengths: $\boldsymbol{\lambda' = \lambda_0(1 - \frac{v}{c})}$

7. The Solar System

7.1 The Solar System

The Sun is the central light source of our Solar System. It is surrounded by 4 inner terrestrial planets, followed by the asteroid belt at 2-3AU, consisting of tens of thousands of small rock/metal/ice asteroids and some minor planets. 4 outer gas giant planets orbit outside this belt, all within 30AU. The Kuiper belt and scattered disk follows from 30-55AU consisting of thousands of frozen volatiles (methane, ammonia, water), comets and notably the dwarf planet Pluto. Even more remote is the hypothesized Oort Cloud.



Recently, Pluto was reclassified as a dwarf planet alongside a number of other discovered objects including Eris (larger than Pluto) and Ceres alongside many other candidates as opposed to classifying all these as conventional planets.

component:	orbital radius: AU	period: yr	mass: M_E (6 10^{24}kg)	radius: R_E (6,378 km)
Mercury	0.39	0.24	0.055	0.383
Venus	0.72	0.62	0.815	0.949
Earth	1.00	1.00	1.00	1.00
Mars	1.52	1.88	0.107	0.533
asteroid belt	1.5-4.0	2-10	0.0006	<0.05
Jupiter	5.20	11.9	318	11.2
Saturn	9.58	29.4	95.2	9.45
Uranus	19.2	83.7	14.5	4.01
Neptune	30.1	163.7	17.1	3.88
Kuiper belt	40-50	$\approx 10^3$	$\approx$0.1	<0.25
Pluto	39	248	0.002	0.19
Eris	68	557	0.003	0.20
Oort cloud	100-50,000	10^4-10^6	$\approx$100	

In the early Solar System the sun was surrounded by a cloud of gas and dust. This collapsed into a disk due to gravity but did not fall into the Sun due to angular momentum. In this disk there were three zones, formed through temperature and density gradiets. In the inner zone, gasses escaped leaving solid, metal rich materials which collapsed into planets. In the middle zone there was lots of ice and good gas condensation/clumping leaving gas/icy giants. In the outer zone there were little collisions however leaving the Kuiper belt.

7.2 Thermodynamics and Photon Pressure

In thermodynamic equilibrium, radiation emitted equals radiation absorbed. This occurs at a specific equilibrium temperature T_{eq}.

$$\boldsymbol{Flux \times Cross\ Section = Surface\ Flux \times Surface\ Area}$$

$$\frac{L_{Sun}}{4\pi d^2} \times \pi R^2 = \sigma {T_{eff}}^4 \times 4\pi R^2$$

Hence: $$\boldsymbol{{T_{eff}}^4 = \frac{L_{Sun}}{16\pi\sigma d^2}}$$

However this is not the end of the story. Planets are not perfect black-body re-emitters. Instead atmospheric dynamics may warm the planet's surface via the greenhouse effect. Nor may a planet be in thermodynamic equilibrium – Mercury for example is hundreds of degrees cooler on its dark side facing away from the Sun.

Comets are icy balls which undergo periodic orbits of the Sun which are highly eccentric. As they near the Sun and heat up they expel gas which provides a large reflective surface for the Sun's light –

bright comet tails. This gas is blown outwards within the solar system due to photon pressure (as the gas/dust grains are so light).

Photons carry momentum, hence exert a pressure: $p = \frac{E}{c}$ $\quad F_{rad} = \frac{dp}{dt}$ $\quad P_{rad} = F_{rad}/Area$

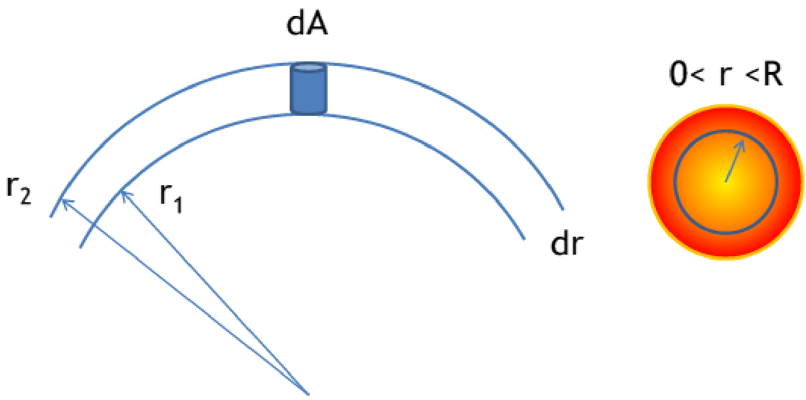
$$F_{rad} = f \times \pi r^2 = \frac{L_{Sun}}{4\pi c d^2} \times \pi r^2 \geq F_{grav} = \frac{GMm}{d^2} = \frac{GM\left(\frac{4}{3}\pi r^3 \rho\right)}{d^2}$$

So as you can see, balancing these forces is independent of distance from the Sun d but instead depends linearly on the radius of the grain r being small enough.

8. The Interior Structure of Stars

8.1 Hydrostatic Balance

Stars need to provide enough pressure at all radii to balance gravity – hydrostatic equilibrium.



Consider a test cylinder as shown with height dr, area dA and mass: $\quad dm = \rho(r) dr dA$

Mass continuity requires: $\quad M(r) = 4\pi \int \rho(r) dr$

It feels gravity: $\quad F_G = GdmM(r)/r^2$

And pressure: $\quad F_P = P(r)dA - P(r + dr)dA = -dPdA$

In equilibrium: $\quad \boldsymbol{\frac{dP}{dr} = -G\rho(r)M(r)/r^2}$

This is the equation of hydrostatic equilibrium and must hold at all $0 < r < R$ otherwise the star would re-adjust.

Consider the entire star as one whole shell:

Central pressure: $\quad P_c = P(r = 0)$

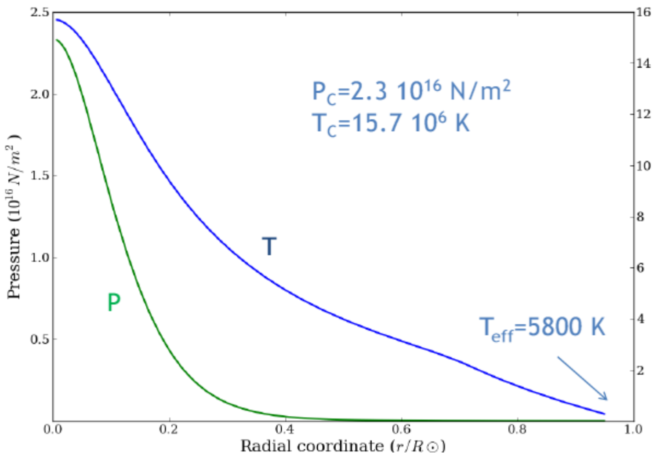
Surface pressure: $\quad P(r = R) = 0$

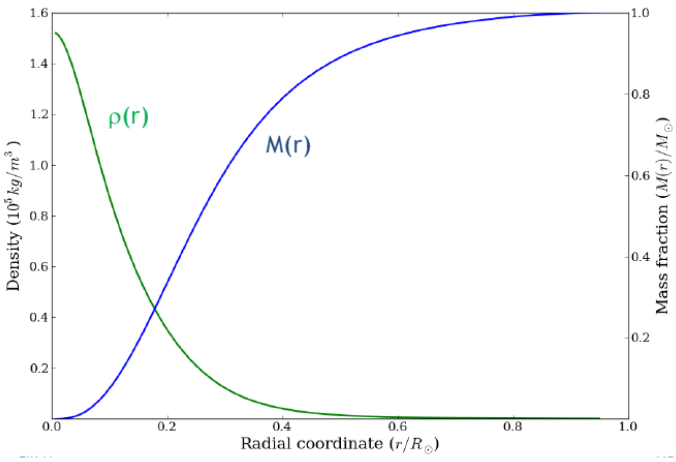
$dp = -P_c \quad dr = R \quad \rho = mean\ density = M(R)/\frac{4}{3}\pi R^3$

So in hydrostatic equilibrium: $\quad \frac{P_C}{R} = \frac{G\rho M(R)}{R^2}$ $\quad \boldsymbol{P_C = \frac{3}{4\pi} GM^2 R^{-4} \propto M^2 R^{-4}}$

We need to know what sets the pressure so we can determine the pressure and density profiles. This information comes from equations of state $P = f(\rho, T \ldots) etc$ Including the ideal gas law:
$P = nkT = \frac{\rho}{\mu} kT$ where n is the number density of particles and μ is the mean mass per particle. For example at the centre of the star, inserting the previous equation for P_c we can show:
$T_C \propto M/R$

Hydrostatic equilibrium basically requires that the thermal energy per particle must equal the gravitational potential energy per particle.

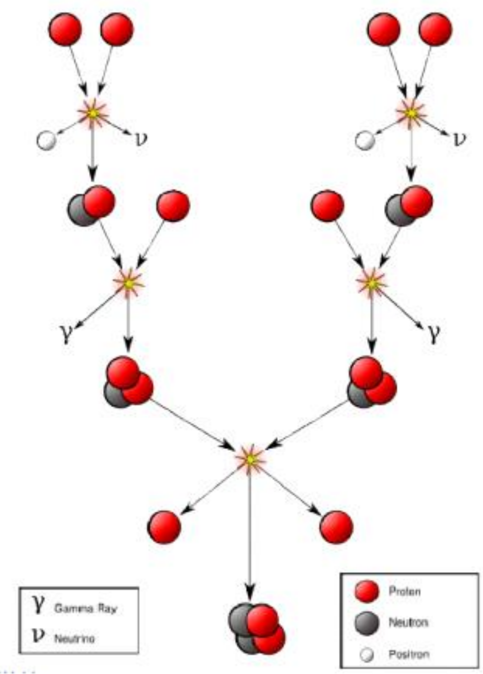




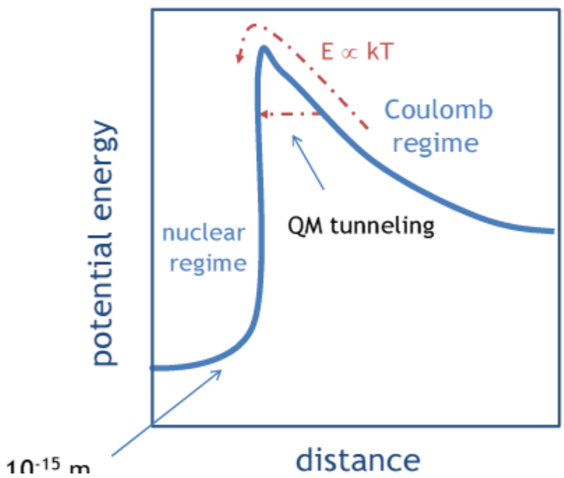
8.2 Energy Production

Hence the star's temperature must provide enough pressure to prevent it from collapsing. This is no small matter as a 15 Million Kelvin solar core for example suffers from huge energy flux losses. Chemical reactions (~13.6eV) are not sufficient to sustain this system. However at 15MK particle nuclei possess enough thermal energy to overcome the coulomb barrier and allow nuclear fusion (with a little help from quantum tunnelling). This process releases massive amounts of mass energy to the order of 26.7MeV in a proton-proton chain (0.7% of mass energy in reactants).

This is enough to sustain a star: $E_{Sun} = 0.7\% \times M_{sun} c^2$ $\quad \frac{E_{Sun}}{L_{sun}} = t \approx 10^{11} yr$



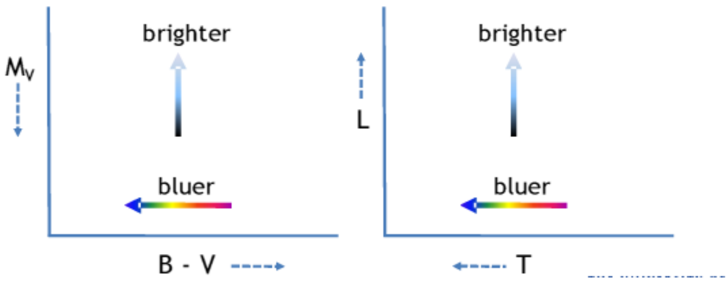
$$4\,{}_1^1H \rightarrow {}_2^4He + 2e^+ + 2\nu_e + 2\gamma$$



9. Hertsprung-Russel Diagrams

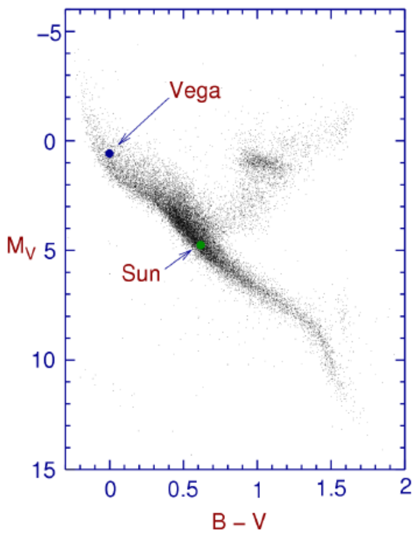
We've seen the structure of stars, and shown that they have incredibly long lifetimes. This poses a problem when trying to comment on their evolution. So instead of observing the process live, we can exploit the family of nearby stars to understand stellar structure and evolution.

Plotting colour/temperature vs luminosity is a Hertsprung-Russel (HR) diagram:



Here: $M_V = m_V - 5log_{10}\left(\frac{d}{10pc}\right)$ is absolute magnitude (don't forget, smaller magnitude=brighter, but it's plotted backwards on a HR!). If L is used it will be on a log scale. If T is plotted, the star will be assumed to be a blackbody.

Hipparchos HR:

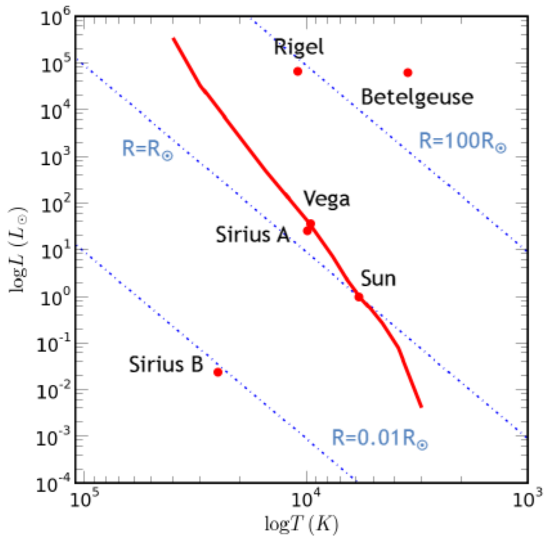


You will notice that the majority of nearby stars form a line running from cool, low luminosity stars to hot, luminous stars – the Main Sequence. As you can see, the Sun is a fairly typical main sequence star.

Recall the Stafan-Blotzmann Law:

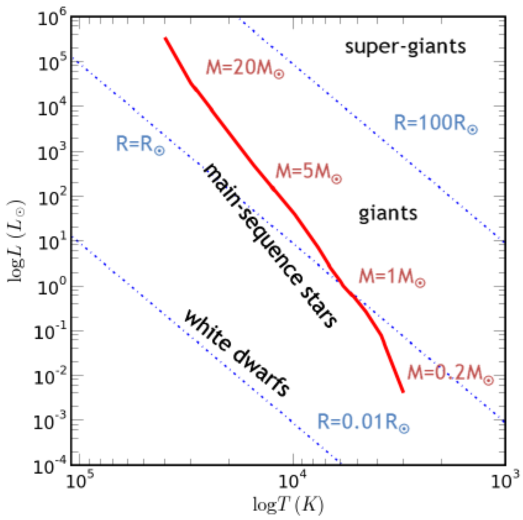
$L = 4\pi R^2 \sigma T^4$

So on a logarthmic scale log(L) vs log(T) is a straight line for constant radius objects.



Some stars like Rigel and Betelgeuse sit above the main sequence and are dubbed giants (over luminous and large). Whereas some sit below such as Sirius B and are dubbed dwarfs (under luminous and small).

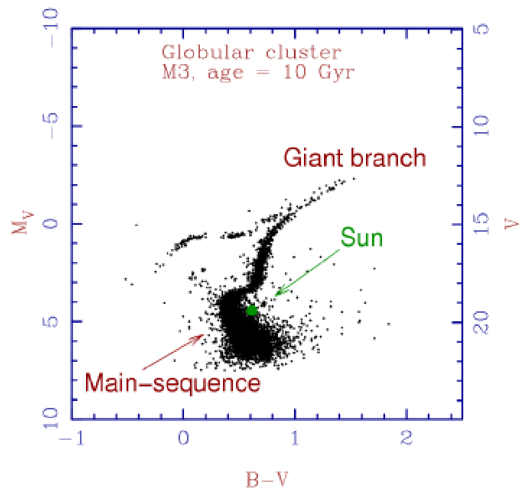
The main sequence implies a tight correlation between a star's luminosity and its temperature (hence radius). This leads to the mass-luminosity relation: $\boldsymbol{L \propto M^4}$
However fuel only scales with M so lifetime of a star: $t \propto M^{-3}$
And large stars live shorter lives. The main sequence is really a mass sequence.



10. Stellar Evolution and End-products

Unfortunately, taking the whole family of nearby stars does not tell us much about their evolution – they are all of different sizes, luminosities and ages etc so are difficult to disentangle. Instead by looking at stellar clusters –stars within a confined region, where all the constituent stars formed from the same constituent gas and are roughly the same age – we can get a better picture of how they evolve from their HR diagrams.

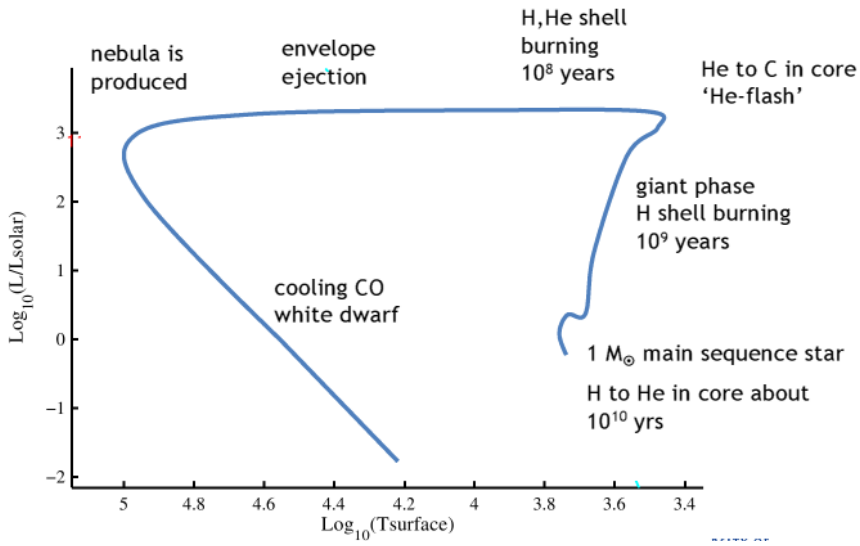
Young stars in a cluster take time to settle onto the bright end of the main sequence (lower mass stars take longer). The cluster then moves along the main sequence and older stars occupy the lower end of the main sequence. By now, the bright end stars have all moved on and stars start to turn off the main sequence onto the giant branch. And finally stars fall off the branch, often into the dwarf region. The older the system the fewer high mass stars there are left.



As we have seen in 8. There are several factors which determine the structure of a star. E(r) energy production, L(r) energy transport, T(r) temperature gradient, M(r) mass continuity, hydrostatic balance and equations of state. However, as the composition of the star changes these have to change and accommodate a new structure solution.

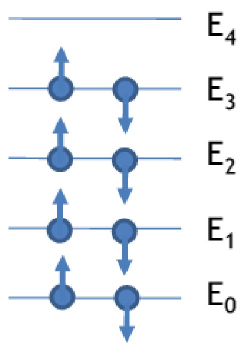
Stars form from clouds of collapsing interstellar gas. The core of this compressing cloud heats up until fusion is possible. On the main sequence H is converted to He in the core of stars, producing a growing He dominated core. There is no fusion in the core which contracts, and H fuses in a shell around the core. In response, the envelope expands and the star leaves the main sequence and becomes a giant. If the core reaches a sufficient temperature then He can begin to fuse to form C+O and the core expands again. This leads to a CO core surrounded by layers of fusing He and H, and a large envelope.

If the giant has insufficient mass to produce temperatures at the core to fuse carbon, then it will now shed its envelope to form a planetary nebula leaving behind the star's core which cools to become a white dwarf.



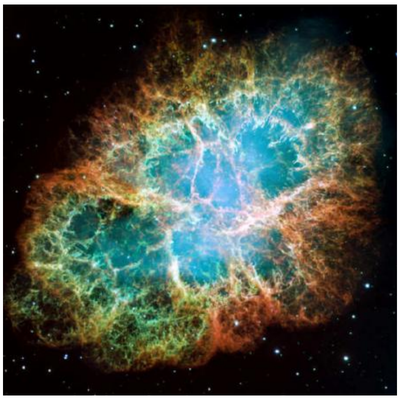
Within a white dwarf, there is no fusion to provide pressure against gravitational collapse. Instead, degenerate electron pressure prevents the white dwarf from collapsing if below the Chandrasekhar Limit.

Electrons are fermions so must obey the Pauli Exclusion Principle – no two fermions can occupy the same quantum level. The numbers of slots are finite so once all the lower levels are filled the only option is for the electrons to occupy the higher ones. Such a fully stacked energy state is called degenerate matter. In such a confined configuration, degenerate matter can exert a much higher pressure than an ideal gas. Pressure in degenerate matter scales with density as ${n_e}^{5/3}$, not with temperature and now degenerate equations of state sets the pressure structure.



At very high densities the electrons become relativistic and pressure grows with ${n_e}^{4/3}$ not with ${n_e}^{5/3}$ which is too slow to provide necessary pressure support. This corresponds to a remnant mass of $M_C = 1.4M_{Sun}$ – the Chandrasekhar Limit.

In massive stars the shell burning/core sequence can extend to heavier elements up to iron. Iron/Nickel have the highest binding energy per nucleon so fusing these nuclei actually costs energy in the core. This energy crisis will lead to a sudden collapse of the core. The ensuing shockwave ejects the envelope in a supernova.



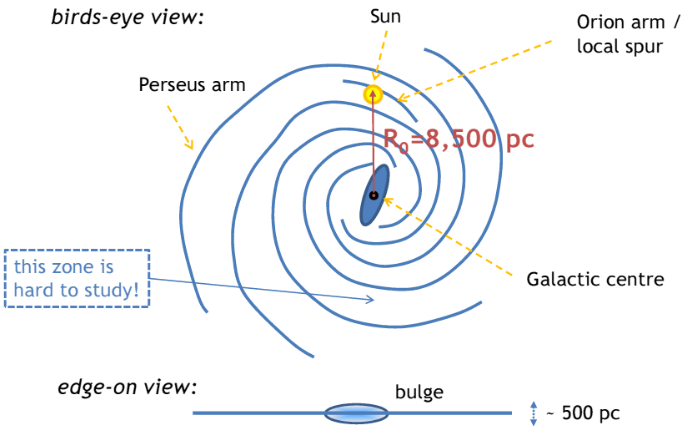
If the mass of the core exceeds the Chandrasekhar Limit then electron degeneracy pressure will not be enough to form a white dwarf. Instead, electrons and protons combine to form neutrons. If the neutron degeneracy pressure is sufficient then the core will form a very compact neutron star.

However, there is again the relativistic mass limit of around $2\ to\ 3M_{Sun}$ beyond which there is no know force to prevent total collapse of the core and forms a black hole.

$M < 0.08M_{Sun}$	Never initiates fusion.
$0.08M_{Sun} < M < 0.8M_{Sun}$	Main sequence lifetime longer than age of Universe.
$0.8M_{Sun} < M < 5\ to\ 8M_{Sun}$	Produce white dwarf.
$5\ to\ 8M_{Sun} < M < 25\ to\ 40M_{Sun}$	Produce neutron star.
$M > 25\ to\ 40M_{Sun}$	Produce black hole.

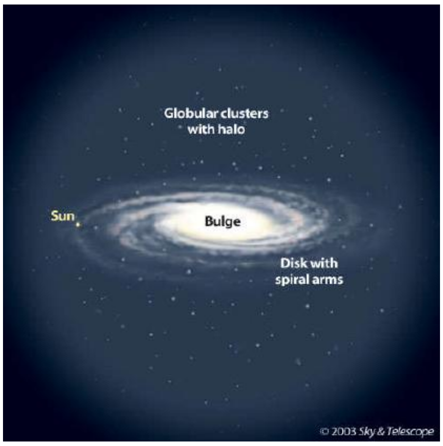
11. The Milky Way and Other Galaxies

The Sun is a typical star in a vast galaxy we call the Milky Way. The Milky Way is a spiral galaxy comprising of a central bulge surrounded by spiral arms in a disk shape. The density of stars peaks in a band running across the sky reflecting our perspective within the disk.



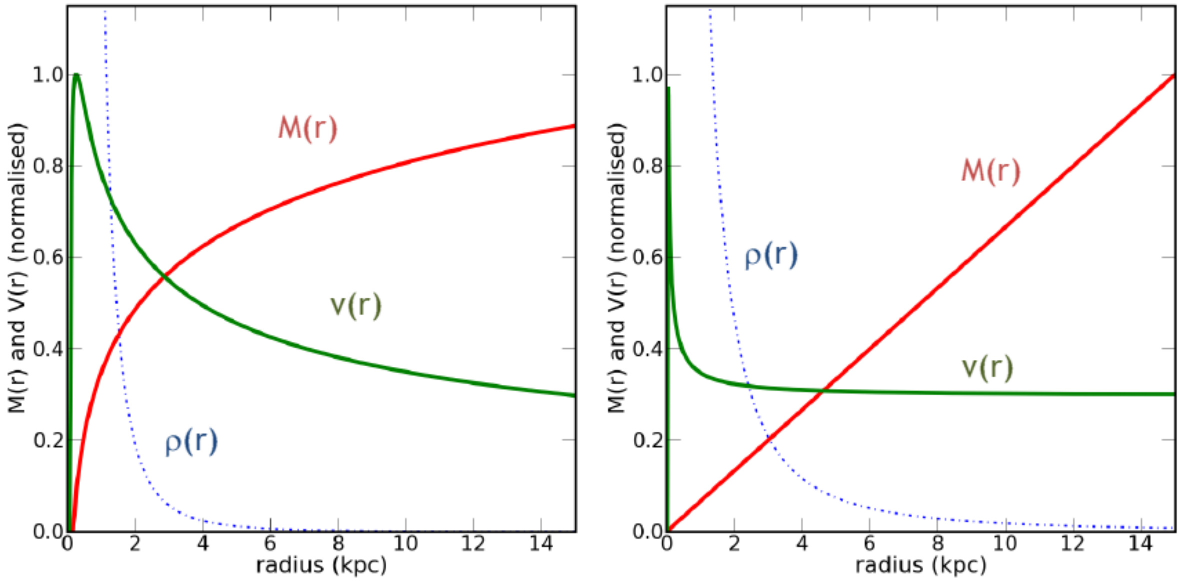
Plotting the distribution of globular clusters on the sky also clearly shows that the Sun is not at the centre of the distribution.

The Sun orbits the centre of the galaxy at about 220km/s, at a radius of 8.5kpc. In theory the bulk movement of mass around the galaxy should balance centripetal and gravitational forces, yielding:



$$M(r) = 4\pi \int x^2 \rho(x) dx = v(r)^2 r/G$$

Where the velocity of stars as a function of distance should track the mass. Since we observe most of the stellar mass to be in the middle, tapering off, this should lead to a dropping $v(r)$ (left). However the observed rotation curve shows a relatively flat $v(r)$ implying $M(r)$ keeps growing! (right)



This flat rotation curve demands that there is non-stellar mass between stars that we have not taken into account – dark matter. This matter is only seen through gravitational interactions as it does not emit any light. All galaxies show this same behaviour implying that the mass in dark matter is greater than that in visible matter. However the nature of dark matter is still a bit of a mystery.

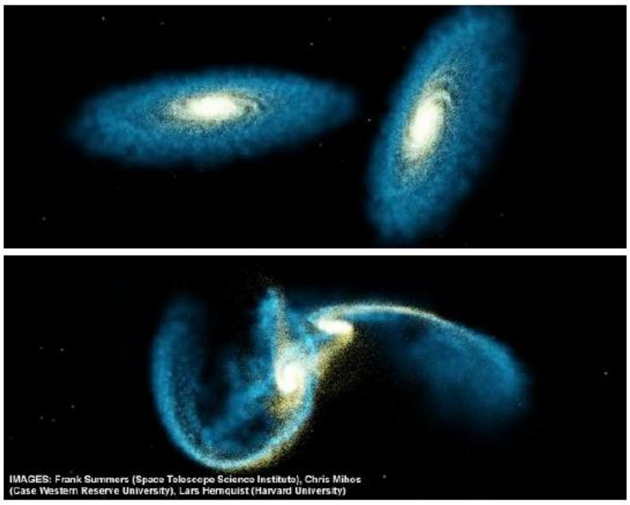
One interesting characteristic of the Milky Way and most (if not all) other galaxies is the existence of a super-massive black hole at the galactic centre. These possess hundreds of thousands to millions of solar masses and too are a bit of a mystery. However given their magnitude it has been suggested that their formation may have interacted with the rotation curve of their entire host galaxy. Super-massive black holes have been detected from the unusual and rapid velocities of stars at galactic centres.

Prior to Hubble, great debate went into whether galaxies were in fact spiral nebulae within our own galaxy, or distinct and distant galaxies. Unfortunately, distance measurements like parallax etc were not appropriate measures. Instead, a standard candle was required, providing M_V from which we could find d from m_V. This came in the form of Cepheids.

Cepheids are giant stars which undergo oscillations in their envelope. Leavitt in 1908 discovered that the period of oscillation of Cepheids correlate with the luminosity of the star: $M_V = -2.81 log_{10} P(d) - 1.43$

Being bright, easily recognisable and with calibrated luminosity they provided the standard candle Hubble needed when he found them in Andromeda in 1923. His observations clearly showed that Andromeda was clearly outside our own galaxy.

Remarkably, the Milky Way and Andromeda are actually on a collision course and will collide in about 2.5 billion years – before the sun becomes a giant.



12. The Evolving Universe

However by measuring the velocities of galaxies using the Doppler shift method, it was shown by Slipher by 1925 that all other galaxies are moving away from us.

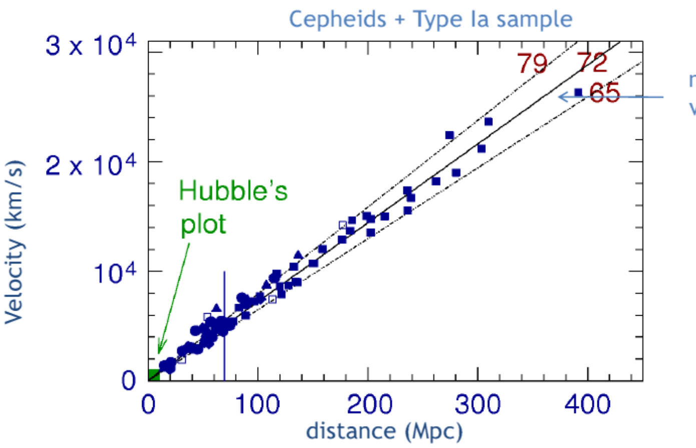
$\boldsymbol{\lambda' = \lambda_0 \left(1 + \frac{v}{c}\right)}$ Redshift: $\boldsymbol{z = \frac{\lambda' - \lambda_0}{\lambda_0} = \frac{v}{c}}$

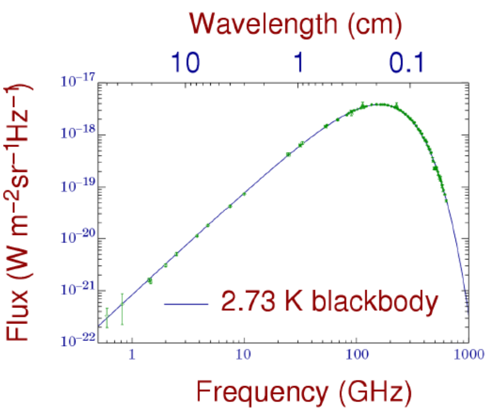
Hubble then noticed a correlation between his distances to galaxies and Slipher's velocities in Hubble's law: $\boldsymbol{v = H_0 d}$ Where $H_0 = 72 kms^{-1} Mpc^{-1}$ is the Hubble constant.

This law implies a centre-less uniform expansion of the universe, and using the current expansion rate we can estimate the age of the Universe: $\frac{1}{H_0} = 1\ Hubble\ Time = 13.6 Gyr$

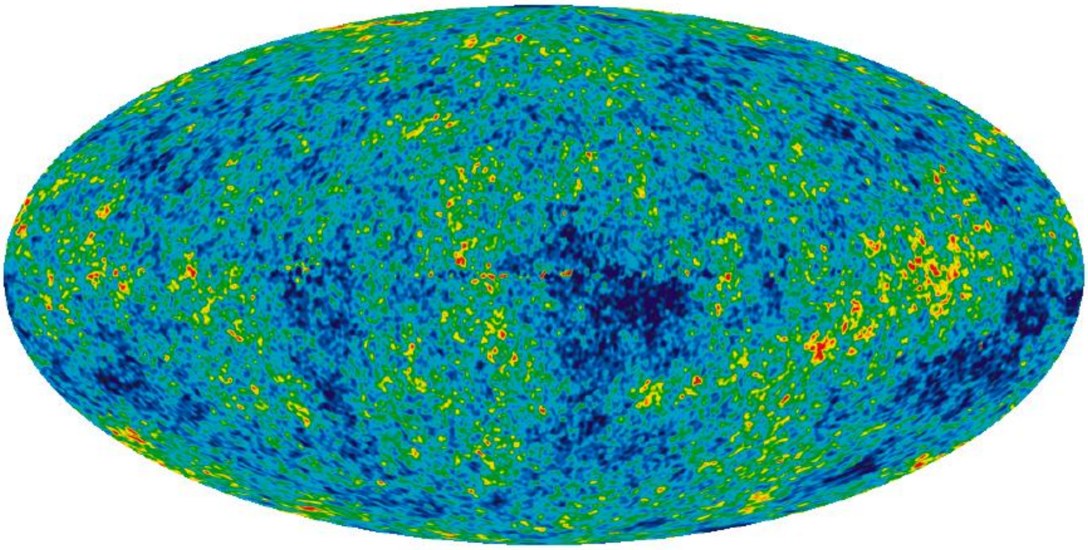
Beginning in a Big Bang.

Cepheids are too faint for us to probe distant galaxies. So distance indicators now include type 1a supernovae, where white dwarfs are pushed over the Chandrasekhar Limit – peaking at roughly the same luminosity with a very distinct light curve - and red shift itself (using Hubble's law to find distance!).





Expansion alone is not the only evidence for the Big Bang. The young universe must have been hot and dense. The Big Bang theory predicts the evidence of this phase should be visible in the present universe in the form of Cosmic Microwave Background – highly red-shifted radiation from the young universe. This was finally detected in 1965 with a blackbody spectrum at 2.7K.



This evidence supports the Big bang theory very well. The false colour above shows only minute differences in CMB ~1/100,000, demonstrating that the universe is quite homogeneous and isotropic.

However, distant supernovae no longer follow the expansion curve we expect. In fact, they are further away than expected and the Universe is accelerating. This requires a repelling term we call dark energy. A lot of dark energy is required to explain the supernovae and CMB constraints, approximately: 4.6% normal matter 23% dark matter 72% dark energy

And this is also a bit of a mystery!