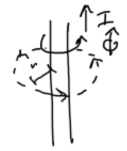
PX120 revision lecture

- Ampère's law
- Biot-Savart law
- Induction

To prepare for exam:
- Problem sheets
- Lecture notes
- Y&F problems and reading

Past paper

Ampère's law: $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ $\qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$



Find $\vec{B}$. $\quad (r,\phi,z) \equiv (\rho,\phi,z)$

Biot-Savart law: $B_z = 0$

Solenoidal cond.: $B_r = 0$

$\Rightarrow \vec{B} = B_\phi \hat{\phi}$

$\vec{B} = B_\phi \hat{\phi} \qquad d\vec{l} = dl\,\hat{\phi}$

$\vec{B}\cdot d\vec{l} = B_\phi \hat{\phi} \cdot dl\,\hat{\phi}$

$= B_\phi\, dl$

$|\vec{B}| = $ constant for fixed radius.

$\therefore \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$

$|\vec{B}| \oint_C dl = \mu_0 I_{encl} = \mu_0 I$

$|\vec{B}|\, 2\pi r = \mu_0 I \Rightarrow |\vec{B}| = \frac{\mu_0 I}{2\pi r} = B_\phi$

Use right-hand rule to find direction.

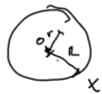
Long, cylindrical conductor of radius R, non-uniform $J = \frac{I}{A} = \alpha r^2$

α = constant

r = radial distance.

Find $\vec{B}$ everywhere.

$I_{encl} = \iint_S \vec{J}\cdot d\vec{s}$



x-section

Exploit symmetry: $B_R = 0$, $B_z = 0$

$\Rightarrow \vec{B} = B_\phi \hat{\phi}$

$B_\phi = \frac{\mu_0 I_{encl}}{2\pi r}$

2 regions: $r < R$, $r \geq R$

$\vec{J} = J\hat{z}$

$d\vec{s} = ds\,\hat{z} = r'\,dr'\,d\phi\,\hat{z}$

$r \geq R$: $I_{encl} = \iint \vec{J}\cdot d\vec{s} = \iint J\, r'\,dr'\,d\phi.$ $\qquad 0 < \phi < 2\pi$, $\quad 0 < r' < R$

$= \int_0^{2\pi}\int_0^R \alpha r'^2 r'\,dr'\,d\phi$

$= 2\pi\alpha \int_0^R r'^3\,dr' = \frac{2\pi\alpha R^4}{4} = \frac{\pi\alpha R^4}{2}$

$B_\phi = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0 \pi\alpha R^4}{4\pi r} = \frac{\mu_0 \alpha R^4}{4r} \quad \therefore \vec{B} = \frac{\mu_0 \alpha R^4}{4r}\hat{\phi}$

$r < R$: $I_{encl} = \iint \vec{J}\cdot d\vec{s} = \iint J r'\,dr'\,d\phi$ $\qquad 0 < \phi < 2\pi$, $\quad 0 < r' < r$

$= \int_0^{2\pi}\int_0^r \alpha r'^2 r'\,dr\,d\phi = 2\pi\alpha\int_0^r r'^3\,dr' = 2\pi\alpha\left[\frac{r'^4}{4}\right]_0^r$

$= \frac{2\pi\alpha r^4}{4} = \frac{\pi\alpha r^4}{2}$

$B_\phi = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0 \pi\alpha r^4}{4\pi r}$ $\quad \vec{B} = \frac{\mu_0 \alpha r^3}{4}\hat{\phi}$

Solenoid, current I, turns per unit length n.

$\vec{B}$ outside $= \vec{0}$ $\qquad 0\hat{x} + 0\hat{y} + 0\hat{z}$

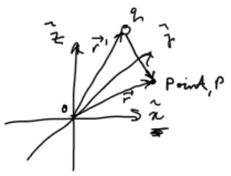
$\vec{B}$ inside $= \mu_0 I n \hat{z}$

Biot-Savart law.

Charge q moving with ~~vel~~ velocity $\vec{v}(t)$, isn't accelerating

+ve or -ve.

$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi} q \frac{\vec{v}(t) \times (\vec{r} - \vec{r}'(t))}{|\vec{r} - \vec{r}'(t)|^3}$$



where: $\vec{r}$ - position of measurement

$\vec{r}'$ - position of charge.

$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{d\vec{r} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

where: $\vec{r}$ - pos. of measurement

$\vec{r}'$ - pos. of element

$d\vec{r}$ - vector line element.

2 examples, one for each form of the law.

① Electron, circular orbit, a_0, velocity v_0.

Find $\vec{B}$ at the origin, $\vec{0}$.

- Use Biot-Savart
- Use cylindrical coordinates

$$\vec{B}(\vec{r},t) = \frac{\mu_0 q}{4\pi} \frac{\vec{v}(t) \times (\vec{r} - \vec{r}'(t))}{|\vec{r} - \vec{r}'(t)|^3}$$

$\vec{v}(t) = v_0 \hat{\phi}$

$\vec{r}'(t) = \vec{r} = a_0 \hat{R}$

~~$\vec{r}$~~ $= \vec{0}$ $\quad q = \underline{-e}$

$\hat{R} \times \hat{\phi} = \hat{z}$

$$= -\frac{\mu_0 e}{4\pi} \frac{v_0 \hat{\phi} \times (\vec{0} - a_0 \hat{R})}{|\vec{0} - a_0 \hat{R}|^3}$$

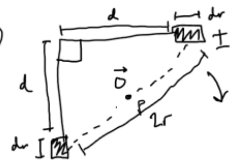
$\underline{\hat{\phi} \times \hat{R} = -\hat{z}}$

$$= -\frac{\mu_0 e}{4\pi} \frac{v_0 \hat{\phi} \times (-a_0 \hat{R})}{|~~-~~a_0 \hat{R}|^3} = ~~-~~\frac{\mu_0 e}{4\pi} \frac{v_0 a_0 ~~-~~\hat{z}}{|a_0 R|^3}$$

$$= \frac{\mu_0 e}{4\pi} \frac{v_0 \cancel{a_0}}{a_0^{\cancel{3}2}} \hat{z}$$

$$= \underline{\frac{\mu_0 e v_0}{4\pi a_0^2} \hat{z}}$$

②

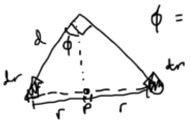


$I = 28\,A$; $dr = \underline{2mm}$; $d = 3cm$.

Find $|\vec{B}|$ at a point P.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{r} \times (\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3} \qquad \vec{r} = \vec{0}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\vec{r} \times ~~-~~\vec{r}'}{|~~-~~\vec{r}'|^3} = \frac{\mu_0 I}{4\pi} \frac{d\vec{r} \times \vec{r}'}{|\vec{r}'|^3}$$



$\phi = 45°$

$|d\vec{r} \times \vec{r}'| = |d\vec{r}| \cdot |\vec{r}'| \sin\phi$

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{dr \cdot \rho \sin\phi}{r^3}$$

$2r = \sqrt{d^2 + d^2} = \sqrt{0.03^2 + 0.03^2}$

$\Rightarrow r = 0.0212$

$$|d\vec{B}| = \frac{4\pi \times 10^{-7} \cdot 28}{4\pi} \frac{2 \times 10^{-3} \cdot \sin 45°}{0.0212^2}$$

$= 8.18 \times 10^{-6}\,T.$

$\Rightarrow |\vec{B}_{total}| = 2 \cdot |d\vec{B}| = \underline{1.64 \times 10^{-5}}\,T$

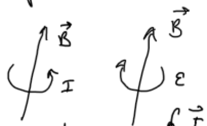
Induction.

- Circuit
- $\vec{E}$ field
- $\vec{B}$ field

Induction

Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S}$

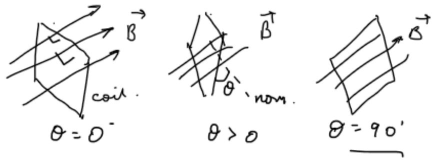
RH-rule defines direction of emf driving a current.



For stationary loops, $\oint_C \vec{E}\cdot d\vec{l} = \mathcal{E} = -\frac{d\Phi_B}{dt}$

(1) Coil with 150 turns, encloses area of 13.1 $\underline{cm^2}$, rotated in 0.042 sec. into a $\vec{B}$-field.
$|\vec{B}| = 7\times10^{-5}$ T.

Starts ⊥ to field, ends // to field.



Φ_B before and after.

$\Phi_B = N\iint \vec{B}\cdot d\vec{S} = N\vec{B}\cdot\vec{S} = NBS\cos\theta = BAN\cos\theta$ (1)

$\Phi_{B,\text{before}} = BAN = 7\times10^{-5}\cdot(13.1\cdot0.01^2)\cdot150$

$= 1.38\times10^{-5}$ Wb. (max. Φ_B)

$\Phi_{B,\text{after}} = BAN\cos\theta = 0$ (min. Φ_B)

$\frac{d\Phi_B}{dt} \approx \frac{\Delta\Phi_B}{\Delta t}$ Sorry mathematicians

Find $\mathcal{E}_{\text{average}}$:

$\mathcal{E} = -\frac{d\Phi_B}{dt} \Rightarrow |\mathcal{E}_{\text{average}}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{1.38\times10^{-5}}{0.042} = \underline{3.29\times10^{-4}\ V}$

BW – 24th May 2021