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1 Constituents of the Standard Model

Standard model of particle physics is a system of relationships between different elementary particles that create all matter and forces.
As it is a model, it does not search for the reason why these specific particles exist, only lists them and leaves free parameters such as their masses and charges to be assigned from measurement.
The basic classification of entities in the standard model is to divide the particles to matter particles and force carriers.

1.1 Matter Particles

There are 12 elementary matter particles in standard model. These can be further divided into quarks, which have colour and interact by strong force, and leptons, which are colourless.
The following table sums up quarks and leptons.

Name	*Symbol*	*Mass* ($\frac{\text{MeV}}{c^2}$)	*Charge* (e)	*Colour*
Up quark	u	2.4	$+\frac{2}{3}$	Yes
Down quark	d	4.8	$-\frac{1}{3}$	Yes
Charm quark	c	1.275×10^3	$\frac{2}{3}$	Yes
Strange quark	s	95	$-\frac{1}{3}$	Yes
Top quark	t	1.72×10^5	$\frac{2}{3}$	Yes
Bottom quark	b	4.18×10^3	$-\frac{1}{3}$	Yes
Electron	e	0.511	-1	No
Electron neutrino	ν_e	≈ 0	0	No
Muon	μ	105.7	-1	No
Muon neutrino	ν_μ	≈ 0	0	No
Tauon	τ	1.78×10^3	-1	No
Tauon neutrino	ν_τ	≈ 0	0	No

It is important to note that the mases of the particles that are created from these particles are not just the sum of the masses of the constituent particles, as there is quite a bit of mass created by the binding energy of the quarks and other constituents.
The charge is the electromagnetic charge, which is a real number. Colour is a changing property of the particle, and has one of three possible values - red, green, blue, with anti-particles having anti-colours anti-red, anti-green and anti-blue. The quarks can be bound only in the states where the overall colour is white, which can be obtained from three particles having red, green and blue colour or two particles with colour and anti-colour pair.
Sometimes, the pentaquark states are also observed, even though very rare.
All matter particles are fermions and they are in fact massless, but mass is produced as a consequence of interaction with the Higgs field, and the mass itself only represents the strength of coupling of the particular particle to the Higgs field.

1.2 Force Carriers

The force carriers are particles that mediate the forces. They are created and absorbed in every interaction of particles, with their lifetime proportional to the inverse of their mass. Therefore, massless particles can mediate interaction without any limiting range, while massive particles have range limit.
The force carrier particles (also called gauge bosons) are summarzized in the table below.

Name	*Symbol*	*Mass* ($\frac{\text{GeV}}{c^2}$)	*Charge* (e)	*Colour*
Photon	γ	0	0	No
Gluon	g	0	0	Colour+anti-colour pair (Not white)
$W^\pm$ boson	$W^\pm$	80.4	± 1	No
Z zero boson	Z^0	91.2	0	No

The strongest force is the strong force. Then, the electromagnetic force, because it has much higher range then the weak force, even though coupling of most particles to the weak field is stronger. The last is the weak force, which is however very important, because it breaks the conservation of flavour, and thus enables more bizzare interactions to occur.

1.3 Higgs Boson

Higgs boson is a boson of mass $\approx 125\,\frac{\mathrm{GeV}}{c^2}$, which interacts with almost all particles to give them their rest mass. This is not the mass that is present in, for example, proton, as most of the mass of the proton is due to the binding energy of the quarks.
Higgs boson is hard to understand and is not covered by this module.

1.4 Common Composite Particles

Some of the most common composite particles (which are always colourless/white) are listed in the table below.

Name	*Symbol*	*Contents*	*Angular Momentum*	*Mass* ($\frac{\mathrm{MeV}}{c^2}$)	*Charge*
Proton	p	uud	0	938	+1
Neutron	n	udd	0	940	0
Pion ±	$\pi^{\pm}$	$u\bar{d}/\bar{u}d$	0	140	±1
Pion zero	π^0	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	0	135	0
Kaon ±	$K^{\pm}$	$u\bar{s}/\bar{u}s$	0	490	±1
Kaon zero	K^0	$\frac{d\bar{s}+\bar{d}s}{\sqrt{2}}$	0	498	0
Delta	Δ	uud	1	1232	+1

1.5 Conservation Laws

Conservation laws help us construct and constraint reactions that can occur between different particles.
This is a quick list of quantitites conserved in particle reactions

- Energy
  - Total energy of a system is always conserved
- Momentum
  - Total momentum of a system is always conserved
- Electric charge
  - Total electric charge is always conserved
- Colour
  - Total colour of the system is always conserved (sum of reds, greens and blues, with anti-colours counting as -1 for given colour)
- Quark flavour (not universal)
  - Number of quarks of given type is conserved, unless the charged weak force bosons mediate the reactions
- Lepton flavour (not universal)
  - Number of leptons of specific type (heavy leptons + neutrinos of that type) is conserved unless charged weak force bosons mediate the reactions
- Baryon number
  - Total number of three quark systems is conserved - required by colour conservation.
- Lepton number
  - Total number of leptons is always conserved

2 Special Theory of Relativity in Particle Physics

2.1 Fourvectors

Fourvectors are ordered quartets of numbers that tranform by Lorentz transformation between different inertial frame. The fourvectors have one temporal component and three spatial components. The Lorentz transformations for four vector **a** are as follows (for inertial frame speed direction in x direction)

$$a'_t = \gamma(a_t - \beta a_x)$$

$$a'_x = \gamma(a_x - \beta a_t)$$

$$a'_y = a_y$$

$$a'_z = a_z$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

and

$$\beta = \frac{v}{c}$$

where v is the speed of the inertial frame in positive x direction, and c is the speed of light.
The Lorentz transformation can be also expressed in the matrix form as

$$\begin{pmatrix} a'_t \\ a'_x \\ a'_y \\ a'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_t \\ a_x \\ a_y \\ a_z \end{pmatrix} = \mathbf{La}$$

Some of the common fourvectors are now briefly introduced. The fourinterval $(ct, x, y, z) = (ct, \vec{r})$, where t is the time of an event, and $\vec{r}$ is the location of an event. The fourmomentum $(\frac{E}{c}, \vec{p})$ is the measure of total energy of the particle.

2.2 Scalar Product

The scalar product for fourvectors is defined as

$$\mathbf{a} \cdot \mathbf{b} = a_t b_t - a_x b_x - a_y b_y - a_z b_z = a_t b_t - \vec{a} \cdot \vec{b}$$

where $\vec{a}$ and $\vec{b}$ are the spatial vectors of the fourvector. This can be also expressed using the so called Minkowski metric as

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_t & a_x & a_y & a_z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} b_t \\ b_x \\ b_y \\ b_z \end{pmatrix} = \mathbf{a}^T \eta \mathbf{b}$$

where the operation implied is the matrix multiplication.
The scalar product is particularly useful because its value is a Lorentz invariant - it is the same for all inertial frames of reference. To see this, consider

$$\mathbf{a}' = \mathbf{La}, \mathbf{b}' = \mathbf{Lb}$$

$$\mathbf{a}' \cdot \mathbf{b}' = (\mathbf{La})^T \eta (\mathbf{Lb}) = \mathbf{a}^T (\mathbf{L}^T \eta \mathbf{L}) \mathbf{b}$$

But, since **L** is a symmetric matrix

$$\mathbf{L}^T \eta \mathbf{L} = \mathbf{L} \eta \mathbf{L} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ \beta\gamma & -\gamma & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \gamma^2 - \gamma^2\beta^2 & 0 & 0 & 0 \\ 0 & \gamma^2\beta^2 - \gamma^2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

From our definition,

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = 1$$

and thus

$$\mathbf{L}^T\eta\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \eta$$

Hence

$$\mathbf{a}' \cdot \mathbf{b}' = (\mathbf{La})^T\eta(\mathbf{Lb}) = \mathbf{a}^T(\mathbf{L}^T\eta\mathbf{L})\mathbf{b} = \mathbf{a}^T\eta\mathbf{b} = \mathbf{a}\cdot\mathbf{b}$$

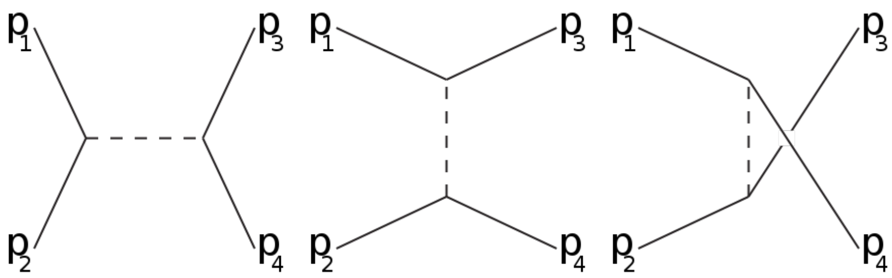
And thus the scalar product is an invariant.
One of more useful quantities that is Lorentz invariant is the Lorentz invariant mass of the system, which is obtained as the magnitude (that is, scalar product with itself) of the fourmomentum

$$\mathbf{p}\cdot\mathbf{p} = \frac{E}{c}^2 - p^2 = \frac{1}{c^2}\left(E^2 - p^2c^2\right) = m^2c^2$$

where m is the rest mass of the object (using Einstains relation for total energy E of an object)

2.3 Mandelstam Variables

Since the total energy in each inertial frame is an additive property and spatial momentum is also additive, we can represent the total fourmomentum of a certain system as the sum of the fourmomenta of the constituents of the system.
Since the total energy and momentum is conserved in each inertial frame, we can say that the fourmomentum of a system is also conserved in each inertial frame. Therefore, the magnitude of this fourmomentum is always conserved and is a Lorentz invariant, which makes it a very useful constant.
The general collision of two particles can occur in one of these three ways.



(a) s-channel (b) t-channel (c) u-channel

In all cases, the magnitude of the sum of the fourmomenta before the interaction must equal the sum of the fourmomenta after the interaction. However, for t and u-channels, additional relationships must apply, leading from the analysis of each vortex - creation and anihilation of mediating particle. For the t-channel, with fourmomentum of the mediating particle labeled as $\mathbf{p}_i$, assuming that the mediating particle emerged from particle 1

$$\mathbf{p}_1 = \mathbf{p}_i + \mathbf{p}_3$$

$$\mathbf{p}_4 = \mathbf{p}_2 + \mathbf{p}_i$$

Hence, by eliminating $\mathbf{p}_i$

$$\mathbf{p}_1 - \mathbf{p}_3 = \mathbf{p}_i = \mathbf{p}_4 - \mathbf{p}_2$$

And thus

$$(\mathbf{p}_1 - \mathbf{p}_3)^2 = (\mathbf{p}_2 - \mathbf{p}_4)^2$$

is a constant in any inertial frame. We could also change the sides of momenta $\mathbf{p}_2$ and $\mathbf{p}_3$ in equation before to get

$$(\mathbf{p}_1 + \mathbf{p}_2)^2 = (\mathbf{p}_3 + \mathbf{p}_4)^2$$

which applies to all channels. Similarly for u-channel

$$\mathbf{p}_1 = \mathbf{p}_i + \mathbf{p}_4$$

$$\mathbf{p}_3 = \mathbf{p}_2 + \mathbf{p}_i$$

$$(\mathbf{p}_1 - \mathbf{p}_4)^2 = (\mathbf{p}_2 - \mathbf{p}_3)^2$$

And this is again compatible with

$$(\mathbf{p}_1 + \mathbf{p}_2)^2 = (\mathbf{p}_3 + \mathbf{p}_4)^2$$

For the s-channel, this is the only equation we can derive, as analysis by vortices gives

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_i = \mathbf{p}_3 + \mathbf{p}_4$$

$$(\mathbf{p}_1 + \mathbf{p}_2)^2 = (\mathbf{p}_3 + \mathbf{p}_4)^2$$

But, in this case, the interacting particle has all the fourmomentum of the system, therefore the highest rest mass.
The magnitude of the fourmomentum of the particle is often denoted by a symbol corresponding to the name of the channel of interaction, i.e. for t-channel

$$t = (\mathbf{p}_1 - \mathbf{p}_3)^2$$

For u-channel

$$u = (\mathbf{p}_1 - \mathbf{p}_4)^2$$

and for s-channel

$$s = (\mathbf{p}_1 + \mathbf{p}_2)^2$$

2.3.1 Two Proton Collision

Assume that two protons are incident on each other with equal and opposite momenta of 6500 $\frac{\text{GeV}}{c}$. As these momenta are much higher than the rest mass energy of the proton, we can say that the total energy of the protons is $\frac{E}{c} \approx \frac{p}{c} = 6500\,\frac{\text{GeV}}{c}$. The s-channel particle has four momentum (using x-axis as the axis of the collision)

$$\mathbf{p}_1 + \mathbf{p}_2 = [(6500, 6500, 0, 0) + (6500, -6500, 0, 0)]\,\frac{\text{GeV}}{c} = (13000, 0, 0, 0)\,\frac{\text{GeV}}{c}$$

hence

$$s = (\mathbf{p}_1 + \mathbf{p}_2)^2 = M^2c^2 = 13000^2\,\frac{\text{GeV}^2}{c^2}$$

where M is the rest mass of the created particle. Hence

$$\sqrt{s} = Mc = 13000\,\frac{\text{GeV}}{c}$$

$$M = 13\,\frac{\text{TeV}}{c^2}$$

2.3.2 Cosmic Rays

Cosmic rays are high energy protons travelling through space. They themselves cannot decay, but they can react with photons of the cosmic microwave background. Assume that a head on collision of proton and photon occurs, labeling E_γ the energy of the photon, E_p the energy of the proton and p the momentum of the proton. Setup the problem so that proton travels in positive x direction and photon in negative x direction.
The momentum of the photon, as it is massless, is $-\frac{E_\gamma}{c}$ in the x direction. Hence

$$s = (\mathbf{p}_1 + \mathbf{p}_2)^2 = \left[\left(\frac{E_p}{c}, p, 0, 0\right) + \left(\frac{E_\gamma}{c}, -\frac{E_\gamma}{c}, 0, 0\right)\right]^2 = \left(\frac{E_p + E_\gamma}{c}\right)^2 - \left(\frac{pc - E_\gamma}{c}\right)^2$$

$$s = \frac{E_p^2}{c^2} + 2\frac{E_p E_\gamma}{c^2} + \frac{E_\gamma^2}{c^2} - p^2 + 2\frac{pE_\gamma}{c} - \frac{E_\gamma^2}{c^2}$$

$$s = \frac{1}{c^2}(E_p^2 - p^2c^2) + 2\frac{E_\gamma}{c^2}(E_p + pc)$$

For very high energy proton, $E_p \approx pc$, hence (also using general relationship $E^2 - p^2c^2 = m^2c^4$)

$$s \approx m_p^2 c^2 + 4\frac{E_\gamma}{c}p$$

The important reaction that can occur is when the centre of mass energy $\sqrt{s}$ is high enough to create a Δ particle. This occurs in case when

$$m_\Delta^2 c^2 = m_p^2 c^2 + 4\frac{E_\gamma}{c}p$$

$$(m_\Delta^2 - m_p^2)c^2 = 4\frac{E_\gamma}{c}p$$

$$p = \frac{m_\Delta^2 - m_p^2}{4E_\gamma}c^3$$

Since photons of microwave background are photons of thermal energy for temperature approximately 2.7 K, their energy is $E_\gamma \approx k_B T \approx 2.3 \times 10^{-4} eV$. Hence

$$p \approx 0.7 \times 10^{12}\ \frac{\text{GeV}}{c}$$

which is effectively the upper limit on the momentum of cosmic protons (any other protons interact with the background and create Δ particles)

3 Basic Interaction of Particles in Matter

3.1 Rutherford Scattering

In Rutherford scattering, a positively charged particle (of charge ze) is incident on a nucleus of charge Ze. We assume that the particle has such energy that the movement of the nucleus can be negeĺected. Assume that the particle arrives along a line with nucleus at distance b from this line.
The line forms effectively the direction of the z axis in the spherical polar coordinate system, with origin at the nuclues. Since we already use z for atomic numbers, I will rename this axis as the x axis, although it works as a z axis. We will assume a cylindrical symmetry (same behaviour for all longtitude angles ϕ).
Because the nucleus is not moving, the potential on the particle is central and hence the angular momentum is conserved. Also, as a consequence of this, the whole motion is realized in one plane. Let θ be the colatitude taken from the direction of the x axis, with increasing x in the direction of movement of the particle.
The magnitude of the angular momentum of the particle at position $\vec{r}$ with speed $\vec{v}$ is

$$L = m|\vec{r} \times \vec{v}|$$

Since the motion is in the plane, $\vec{\omega} \perp \vec{r}$ and hence

$$L = -m\omega r^2$$

where $\omega = |\vec{\omega}| = \frac{d\theta}{dt}$ and $r = |\vec{r}|$. The minus sign appears because at the beginning, the momentum L is pointing in the negative direction (θ decreases as time passes).
Hence

$$-\omega r^2 = |\vec{r} \times \vec{v}|$$

At the beginning of the movement, for particle at distant approach to nucleus at distance $x = d >> b$, $\vec{r} = (-d, b, 0)$ and $\vec{v} = (v_0, 0, 0)$, and therefore

$$\vec{r} \times \vec{v} = (0, 0, -bv_0)$$

$$|\vec{r} \times \vec{v}| = bv_0$$

and therefore

$$r^2 = -\frac{bv_0}{\omega} = -\frac{bv_0}{\frac{d\theta}{dt}}$$

The force in the direction perpendicular to the movement (call it the y direction) is

$$F_y = m\frac{dv_y}{dt} = ze\frac{Ze}{4\pi\epsilon_0 r^2}\sin\theta = -\frac{zZe^2}{4\pi\epsilon_0 bv_0}\sin\theta\frac{d\theta}{dt}$$

Changing the derivatives

$$\frac{dv_y}{d\theta} = -\frac{zZe^2}{4\pi\epsilon_0 bmv_0}\sin\theta$$

Hence

$$v_y(\theta) = C + \frac{zZe^2}{4\pi\epsilon_0 bmv_0}\cos\theta$$

where C is integration constant. For $\theta \to \pi$, $v_y \to 0$ so

$$0 = C + \frac{zZe^2}{4\pi\epsilon_0 bmv_0}\cos(\pi)$$

$$C = \frac{zZe^2}{4\pi\epsilon_0 bmv_0}$$

So

$$v_y(\theta) = \frac{zZe^2}{4\pi\epsilon_0 bmv_0}(1+\cos\theta)$$

As the nucleus does not move, the scattered particle at big distance from the nucleus must have the same energy as the incident particle, and therefore also the same speed. Hence the final velocity of the particle is

$$\vec{v}_f = (v_0\cos\theta_f, v_0\sin\theta_f, 0)$$

Where θ_f is the final outgoing angle of the particle. Using the equation for v_y at θ_f

$$\frac{zZe^2}{4\pi\epsilon_0 bmv_0}(1+\cos\theta_f) = v_0\sin\theta_f$$

$$\frac{\sin\theta_f}{1+\cos\theta_f} = \frac{zZe^2}{4\pi\epsilon_0 bmv_0^2}$$

But

$$\frac{\sin\theta_f}{1+\cos\theta_f} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

Now, we would like to determine standard quantity that represents scattering of particles, the differential cross-section. Differential cross-section corresponds to the ratio of an area $d\sigma$ from which the particles are scattered into the solid angle $d\Omega$ at some direction θ. We can see that the direction θ only depends on the initial distance from the nucleus b and velocity squared of the particle v_0^2. Hence we can parametrize the are $d\sigma$ as

$$d\sigma = bd\phi db$$

where ϕ is the lattitude angle.
The solid angle element in direction θ is

$$d\Omega = \sin\theta d\theta d\phi$$

Since scattering is independent of ϕ, we can than state

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta}$$

And

$$b = \frac{zZe^2}{4\pi\epsilon_0 mv_0^2\tan\frac{\theta}{2}}$$

$$\frac{db}{d\theta} = \frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\frac{-1}{\tan^2\frac{\theta}{2}}\frac{1}{\cos^2\frac{\theta}{2}}\frac{1}{2} = -\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\frac{1}{2\sin^2\frac{\theta}{2}}$$

Hence

$$\frac{d\sigma}{d\Omega} = -\frac{b}{\sin\theta}\frac{zZe^2}{8\pi\epsilon_0 mv_0^2\sin^2\frac{\theta}{2}}$$

Substituting for b

$$\frac{d\sigma}{d\Omega} = -\frac{1}{2\sin\theta}\frac{1}{\tan\frac{\theta}{2}}\frac{1}{\sin^2\frac{\theta}{2}}\left[\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\right]^2 = -\frac{1}{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\sin^2\frac{\theta}{2}}\left[\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\right]^2 = -\frac{1}{4\sin^4\frac{\theta}{2}}\left[\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\right]^2$$

The negative sign represents the fact that smaller area σ corresponds to scattering into bigger angles θ. If we are, however, only interested in the magnitude, we can use

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\sin^4\frac{\theta}{2}}\left[\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}\right]^2$$

Interestingly, the closest approach possible by a particle is when the particle approaches head on to the nucleus. It then approaches up to distance D, at which

$$\frac{zZe^2}{4\pi\epsilon_0 D} = \frac{1}{2}mv_0^2$$

(all energy as potential energy)
then

$$D = 2\frac{zZe^2}{4\pi\epsilon_0 mv_0^2}$$

And therefore

$$\frac{d\sigma}{d\Omega} = \frac{D^2}{16\sin^4\frac{\theta}{2}}$$

This means that we can estimate the size of the nucleus as D by firing particles at it. This was indeed what Rutherford have done.

3.2 Ionization Losses

Particles in matter can lose energy by many processes, with most dominant being pair/particle creation interactions and ionization losses.
Ionization losses occur by charged particles moving through a matter and ionizing electrons of the atoms of the matter. To find the energy given to some electron at distance b from linear trajectory of the incoming particle, we must make several assumptions.
First, we assume that the particle is so fast that it does not change its direction of movement by the interaction with electron very much. Second is that the interaction between the electron and the particle occurs in such a small time that the electron only effectively starts moving after the interaction is over. Last assumption is that the potential and kinetic energy of the electrons are negligable.
With these assumptions, we have a particle with charge ze incoming on a stationary free electron with charge e.
The y component of the force acting on the electron is

$$F_y = \frac{ze^2}{4\pi\epsilon_0(x^2+b^2)}\sin\theta$$

where x is the distance of the particle from the closest approach to the electron, when from electron to particle is b (and x is therefore 0, and is therefore the origin), and θ is the angle between the vector pointing from the electron to the particle.
$\sin\theta$ can be also reexpressed as

$$\sin\theta = \frac{b}{\sqrt{x^2+b^2}}$$

Hence

$$F_y = \frac{bze^2}{4\pi\epsilon_0(x^2+b^2)^{\frac{3}{2}}}$$

The momentum gained by the electron in the y direction is then

$$p_y = \int_{-\infty}^{\infty} F_y dt = \int_{-\infty}^{\infty} F_y \frac{dt}{dx} dx = \frac{1}{\frac{dx}{dt}} \int_{-\infty}^{\infty} \frac{bze^2}{4\pi\epsilon_0 (x^2+b^2)^{\frac{3}{2}}}$$

Here $\frac{dx}{dt} = v_0$ is the initial velocity of the particle. Hence

$$p_y = \frac{bze^2}{4\pi\epsilon_0 v_0} \int_{-\infty}^{\infty} \frac{1}{(x^2+b^2)^{\frac{3}{2}}} dx$$

The integral evaluates as (using $b\alpha = x$)

$$I = \int_{-\infty}^{\infty} \frac{1}{(x^2+b^2)^{\frac{3}{2}}} dx = \int_{-\infty}^{\infty} \frac{1}{b^3(1+\alpha^2)^{\frac{3}{2}}} b d\alpha = \frac{1}{b^2} \int_{-\infty}^{\infty} \frac{1}{(1+\alpha^2)^{\frac{3}{2}}} d\alpha$$

Using another substitution $\alpha = \tan\phi$

$$I = \frac{1}{b^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{(1+\tan^2\phi)^{\frac{3}{2}}} \frac{1}{\cos^2\phi} d\phi = \frac{1}{b^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\frac{1}{\cos^3\phi}} \frac{1}{\cos^2\phi} d\phi = \frac{1}{b^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\phi d\phi = \frac{1}{b^2} [\sin\phi]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{2}{b^2}$$

Hence

$$p_y = \frac{ze^2}{2\pi\epsilon_0 v_0 b}$$

The force in the x direction is

$$F_x = \frac{ze^2}{4\pi\epsilon_0(x^2+b^2)} \cos\theta = \frac{ze^2}{4\pi\epsilon_0(x^2+b^2)} \frac{x}{\sqrt{x^2+b^2}} = \frac{x}{b} F_y$$

Hence

$$p_x = \frac{ze^2}{4\pi\epsilon_0 v_0} \int_{-\infty}^{\infty} \frac{x}{(x^2+b^2)^{\frac{3}{2}}} dx$$

The integral is now

$$I = \int_{-\infty}^{\infty} \frac{x}{(x^2+b^2)^{\frac{3}{2}}} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{b\tan\phi}{(b^2\tan^2\phi + b^2)^{\frac{3}{2}}} \frac{b}{\cos^2\phi} d\phi =$$

$$= \frac{1}{b} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\phi \tan\phi d\phi = \frac{1}{b} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\phi d\phi = \frac{1}{b} [\cos\phi]_{\frac{\pi}{2}}^{-\frac{\pi}{2}} = 0$$

Therefore, the momentum is only transfered in the y direction, which corresponds to our assumptions (particle moves so fast that equal momentum from incident and emergent particle is trasfered in opposite directions to the electron).
The energy transferred to the one electron is then

$$E_1 = \frac{p^2}{2m_e} = \frac{p_y^2}{2m_e} = \frac{z^2e^4}{8\pi^2\epsilon_0^2 m_e v_0^2 b^2}$$

where m_e is the mass of the electron.
To calculate the total energy deposited in the material, we need to integrate over all reasonable radii b and include loss due to all electrons. Let the trajectory of the particle be the x axis as before and lets define cylindrical coordinate system with radius b in the yz plane and angle ϕ in the same plane and x as the third, cartesian coordinate. The total number of electrons in a cylindrical shell of height dx and radius b and width db is

$$dN = 2\pi b db dx n$$

where n is the density of the electrons.
The energy transferred to these electrons is

$$dE = E_1 dN = \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e v_0^2 b} db dx$$

And the total energy deposited per length dx is then

$$\frac{dE_T}{dx} = \int_{b_{min}}^{b_{max}} \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e v_0^2 b} db$$

$$\frac{dE_T}{dx} = \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e v_0^2} \int_{b_{min}}^{b_{max}} \frac{1}{b} db = \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e v_0^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

where $b_{min/max}$ are some artificial limits. For our purposes, the limits will be defined by two considerations - first is that if the electrons orbit too fast, they do not interact as we assumed. For frequency ν_e of the electrons in orbit, their time period, as seen from the perspective of the particle, is

$$T' = \gamma T = \gamma \frac{1}{\nu_e}$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ (and $\beta = \frac{v_0}{c}$) is the Lorentz factor of the particle. If this time is about the same as the time the particle does most of the interaction (in distance about b from the electron, hence while travelling distance about b around the point of closest approach), the interaction is problematic and starts to be weak (bondign effects are stronger). The time for the particle to travel b is

$$T_p = \frac{b}{v_0}$$

Hence

$$\frac{b_{max}}{v_0} = T_{p,max} = T' = \gamma \frac{1}{\nu_e}$$

$$b_{max} = \gamma \frac{v_0}{\nu_e}$$

The second consideration is the quantum mechanical nature of the electron, which probably obscurs the interaction as well. This probably occurs at distance b_{min} on which the wave properties of the electron become dominant, which means that the wavelength of the electron is about $\lambda_e \approx b_{min}$.
In the frame of reference of the particle, the electron is moving towards the particle at speed v_0 and thus has relativistic momentum

$$p_e = \gamma m_e v_0$$

Hence the wavelength of this electron is (using de Broglie relation)

$$\lambda_e = \frac{2\pi\hbar}{p} = \frac{2\pi\hbar}{\gamma m_e v_0}$$

Hence

$$b_{min} \approx \lambda_e = \frac{2\pi\hbar}{\gamma m_e v_0}$$

And therefore

$$\frac{dE}{dx} = \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e v_0^2} \ln\left(\frac{\gamma^2 m_e v_0^2}{2\pi\hbar\nu_e}\right)$$

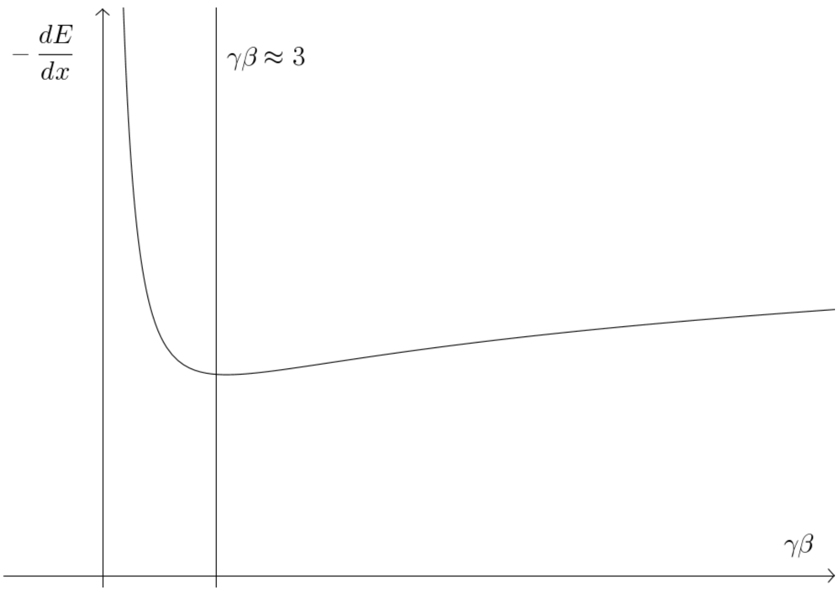
This energy is lost from the particle. This means that if we change the meaning of E to the energy of particle, the energy loss is (also rewritting everything in relativistic β)

$$-\frac{dE}{dx} = \frac{nz^2e^4}{4\pi\epsilon_0^2 m_e c^2} \frac{1}{\beta^2} \ln\left(\frac{\gamma^2\beta^2 m_e c^2}{2\pi\hbar\nu_e}\right)$$

This is our very approximate form. The Particle Group version of this formula is

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2m_e c^2\gamma^2\beta^2 W_{max}(\gamma^2\beta^2)}{I^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

For us, the exact form is not that important, important are only the qualitative analysis as follows for the original formula.
We see that for high energies, $\beta \to 1$ and the dependence is purely $-\frac{dE}{dx} \propto \ln(\gamma^2) \propto \ln\gamma$. But, for smaller energies, the β parameter starts to vary and the $\frac{1}{\beta^2}$ dependence starts to matter. The overall behaviour is similar to one displayed in figure below.
The specific curves vary for each particle and material, but the important feature is that the minimum ionizing energy occurs at about $\gamma\beta \approx 3$.
For particles slower than this, the ionization losses start to rise quickly and particle quickly stops afterwards.



3.2.1 Delta rays

If the incident particle is very fast and approaches the electron at very small b, it can eject the electron at such speed that the electron itself becomes the fast moving particle. These secondary particles are called the Δ rays (not to confuse with Δ particles, which are a different type of particle).
Delta rays can be usually recognized as small trajectories of highly curving (i.e. very light) particles beginning at the major particle trajectory.

3.2.2 Bragg Peak

Since the energy deposit of the particle first slowly decreases and afterwards rapidly increases with decreasing energy of the particle, the energy deposit peaks right before the particle stops.
This peak is called the Bragg peak, and the top intensity at the Bragg peak is usually about 10 times more than at minimum ionizing (at $\gamma\beta = 3$).
The particle then comes to stop. This can be used for controlled depth of deposit of energy - the energy of accelerated particles is tuned so that they deposit most of their energy only at specific depth into the material, where the Bragg peak occurs.

3.3 Thick Materials

3.3.1 Brehmstralung

For thick materials or very fast particles, other type of energy dissipation becomes dominant - the brehmstralung - braking radiation. The particle interacts with other charged particles and in the process emits a photon. As the energy of the particle is very high, this photon can become real photon with high energy that can then split into an electron-positron pair, which react further.
Usually, the energy is approximately equally distributed among the resultant particles, and the length which particles travel between each emmision is approximately constant as well. This means that the overall decrease in energy is exponential in form of

$$E = E_0 e^{-\frac{x}{L_R}}$$

where L_R is called the radiation length. The radiation length is usually smaller for lighter particles (depends on the square of the mass of the particle $L_R \propto m^2$). Therefore, the electrons are the strongest interacting particles, muons less etc.
The energy of the particle when the brehmstralung becomes important vary depending on a mass, but for electron, this critical energy is 10-100 MeV, for muon, it is about 100-1000 GeV (muon is about hundred times more massive, hence the energy is about 10 000 times higher).
Several remarks need to be made. On average, the length between photon splitting into e^+e^- pair (pair creation) is slightly longer, so we have

$$L_{pp} \approx \frac{9}{7} L_R$$

Also, it usually makes sense to define the radiation length dependent on a material, but not on the material density. Then we have specific radiation length

$$X_0 = L_R\rho$$

which is not dependent on the density (while L_R is). Therefore, we can write

$$L_R = \frac{X_0}{\rho}$$

3.3.2 Hadronic Interaction

Hardons (particles consisting of three or more quarks) react in the thick matter also due to the interactions of the strong force with the nuclei. However, since nuclei are so small, the hadronic interaction length for this interaction is about 10 times of the radiation length in the same material, i. e.

$$L_I \approx 10L_R$$

and the energy goes as

$$E = E_0 e^{-\frac{x}{L_I}}$$

Since hadrons are usually much more massive, the hadronic interaction becomes their main source of energy loss, overweighting the radiative losses.
During the interaction, multiple mesons (more than one or pair) can be created, and the hadron can transform. So, the hadronic interactions are generally much more messy than the electromagnetic radiative interactions. Also, hardons can sometimes change into neutrons, which usually escape detection, as they do not end up ionizing as they stop. In practice this means that only about 40 % of the initial energy is deposited in the material in form of ionization, compared to nearly all the energy in the case of electromagnetic radiation.

3.3.3 Multiple Scattering

When the material is thick, the beam of particles can be scattered multiple times. Since the derivation from Rutherford scattering is hard and only approximate, we use central limit theorem to model the resulting distribution of angles as Gaussian, i. e.

$$\frac{d\sigma}{d\Omega} \propto e^{-\frac{(\theta-\theta_0)^2}{2\sigma_\theta^2}}$$

where θ_0 is the original direction of the beam and σ_θ is the standard deviation parameter. The standard deviation is usually modeled as

$$\sigma_\theta \approx 13.6\,\mathrm{MeV}\frac{z}{\beta cp}\sqrt{\frac{x}{L_R}}\left(1 + 0.38\ln\left(\frac{x}{L_R}\right)\right)$$

where z is the atomic number of the particle (number of positive e charges), p is the momentum of the particle, β and c have usual meanings and x is distance travelled in the medium and L_R is the radiation length in the medium.

4 Detectors

4.1 Detected Particles

Out of all possible particles that can be created, only a handful is usually observable in the detector. This is because other particles are usually not stable and decay by electromagnetic or strong force, and thus decay right after the collision, before reaching any detector. A few particles that are stable and observable are

- Photon, γ
  - Photons are not directly observable, but they can cause ionization and at sufficient energies can split into electron - positron pair, causing what is known as electromagnetic showers
- Electron/positron, $e^{\mp}$

  - Quickly curves in magnetic fields and causes electromagnetic showers

- Muon/Anti-muon, $\mu^{\mp}$
  - Heavier but otherwise similar to electron, these particles do not curve nearly as much as electrons and also do not usually cause electromagnetic showers until they slow down a lot, and by that time they usually decay (by weak force to electrons/positrons)

- Tauons/anti-tauons, $\tau^{\mp}$
  - Usually decay before reaching detector, but sometimes can enter at very high energies of the colliding particles.

- Proton, p
  - Very stable and massive, protons usually do not cause electromagnetic showers, but they do interact by hadronic interaction.

- Neutron, n
  - Neutrons cannot be directly seen in the detectors, as they do not ionize, but they react by hadronic interactions, which can create hadronic showers (analogue of EM showers)

- Pions, $\pi^{\pm}, \pi^{0}$
  - Charged pions decay by weak force, and therefore can appear in the detector. They are lighter than protons and heavier than positrons, so they can usually be distinguished from these. However, they can be mistaken for muons. To distinguish these, muons do not react by hadronic interactions, while pions do. Charged pions usually decay to muons.
  - Neutral pions decay by EM force, so they usually do not reach the detector, but at very high energies they can. The decay length of neutral pion (0.3 nm) is the shortest measured decay length. Neutral pions usually decay to photons.

- Kaons, $K^{\pm}, K^{0}$
  - Similar behaviour as pions, but they have one strange quark instead of the down quark, making them less stable.

4.2 Observable Interactions

The interactions we observe in detectors can be split into three categories - electron/positron interactions, low energy hadronic interactions and high energy hadronic interactions.

4.2.1 Electron/positron Interactions

Electrons and positrons create both photons and start electromagnetic showers - the secondary photons split into more electrons and positrons and the cycle repeats until the energy is depleted. But, when collided, positron and electron can anihilate, which can cause an off shell momentum photon to form and then split into colourless quark - anti-quark pair. This creates hadronic jets emergent even from quark-less collisions.

4.2.2 Low Energy Hadronic Interactions

Often creates light mesons (Kaons or Pions) by strong interaction, but not many other particles, as there is not enough energy for their creation. Can cause nuclear decay, and are sometimes used to create a good neutron source.

4.2.3 High Energy Hadronic Interactions

Any kind of strong interaction can take place, leading to strong hadronic showers, many jets and many new particles. Curiously, when colliding protons, the cross-section of any reaction is very small, and have to take a lot of data, otherwise the protons just recoil and then cause high energy interactions without any new particle formed.

4.3 Tracking Detectors

The tracking detectors try to measure the path of the particle inside it ideally without disturbing the particle. From the path of the particle, information about the particle can be deduced.
Usually, tracking detectors try to measure path of charged particles in magnetic fields, which have well defined behaviour (discussed later) and also usually cause ionisation, which is an easy way how to track the movement of the particle. The only exception to this are the detectors using Cherenkov radiation, which can in principle detect directly photons or other particles, but this is not discussed in this course.

4.3.1 Photographic Film/Emulsion

In early days of particle physics, photographic films were used to track the particles - a big block of AgI solid (called emulsion) was placed in a magnetic field on top of a mountain for several days, and afterwards carefully sliced. The ionization caused by the particles worked in the same way as for photography - the tracks turned black.
Emulsions are still one of the most precise tracking devices, but the collection and reset times for these are immensely long.

4.3.2 Bubble Chamber

Bubble chamber uses superheated liquid that creates bubbles when ionization occurs. The ionized electrons act as condensation centres and cause the superheated liquid to boil. The bubble size can be controlled by the pressure in the liquid and thus the bubble chamber can be well calibrated using known sources of particles with stable energy (such as radioactive decay with 7 MeV α particles etc.).
The liquid needs to be again as light as possible to prevent from unwanted interactions with the particles. Therefore, liquid hydrogen is often used, which is quite dangerous and usually requires preventive measures to be taken.
Otherwise, the bubble chamber is much faster than the emulsion and can be made much bigger and even automatic - automatic photos taken at some time interval. This means that bubble chamber needs very good optics. The Delta rays can be used to check the direction of a magnetic field and thus find the charge of the particle/direction of movement of the particle.
The typical operation of bubble chamber is up to frequency of some Hz.

4.3.3 Wire Chamber

Wire chamber uses gas ionization which emits electrons. These electrons are then collected by the positively charged wires which run in the chamber. This means that the ionization can be pretty directly measured and directly transferred to electronic signal. This enables wire chamber to have measurement frequency of several MHz, which is probably the fastest collection time for relatively continuous track information.
The designs of wire chamber vary but they usually need to be big because the electron response for a particle is small (not many electrons generated in the gas by its ionization), hence the precision must be gained by particle travelling for relatively long time. But, since the gas is so light, little to no scattering and brehmstrahlung occur in this detector. The resolution is similar to the bubble chamber, only slightly worse.

4.3.4 Silicon Detectors

Silicon detectors use silicon diodes that are under voltage just before breakdown. When a particle ionizes the silicon inside the diode, electron-hole pair is created and accelerated towards the ends of the diode, creating a signal.
The resolution of silicon detectors is usually about 20 - 30 μm, which is about 10 times worse than that of bubble/wire chamber, but the collection time is very fast (about 40 MHz), slightly more than wire chamber, and the silicon chips are relatively cheap.
But, because silicon is dense, layered structure needs to be used to achieve tracking without scattering.
There are two geometries for the layers commonly used - strip and pixel detectors. Strip detectors use array of long strip diodes to collect the data, thus only receiving one dimension of the direction of movement. Pixel detectors detect in both directions, but use much more cables, which reduces the precision of the experiment and also is expensive. Usually, two strip detectors, perpendicular in strip direction to each other, are rather used ($2L$ of wires instead of L^2 of wires).
The response density (number of electrons ionized per particle) is good, around 2300 e^- per particle in common detectors (about 300 μm thick with diode separation about 80 μm).

4.4 Momentum and Direction Determination in 2D

We now present a few derivations for the error in momentum and direction of a particle in 2D detector - usually composed of multiple one layer detectors.
Lets start with the direction. The direction of particle in 2D can be determined by the angle θ between the trajectory of the particle and the detection plane of the strip detector. For two points in two consequent strip detectors separated by distance L and measuring position in the x direction, the angle is

$$\theta = \tan^{-1}\left(\frac{D}{L}\right)$$

where $D = x_2 - x_1$ and $x_{1/2}$ are the positions measured by detector 1/2.
Usually, the particles move very fast and almost normal to the plane of detection, so

$$\tan^{-1}\left(\frac{D}{L}\right) \approx \frac{D}{L}$$

Therefore, the error of direction due to uncertainty in x_1 is

$$\Delta\theta_1 \approx \frac{\partial\theta}{\partial x_1}\Delta x_1 = -\frac{\Delta x_1}{L}$$

and

$$\Delta\theta_2 \approx \frac{\partial\theta}{\partial x_2} = \frac{\Delta x_2}{L}$$

if we assume that the detectors are the same with $\Delta x = \Delta x_1 = \Delta x_2$, the overall error in θ due to detector uncertainty is

$$\sigma_{\theta,D} \approx \sqrt{\Delta\theta_1 + \Delta\theta_2} = \sqrt{2}\frac{\Delta x}{L}$$

But, we have also the scattering term $\sigma_{\theta,S}$, which decreases with the momentum of the particle. The overall error in the direction is therefore

$$\sigma_\theta \approx \sqrt{\sigma_{\theta,D}^2 + \sigma_{\theta,S}^2}$$

And is very high for slow particles, which scatter a lot, but reaches $\sigma_{\theta,D}$ in limit $p \to \infty$, because there $\sigma_{\theta,S} \to 0$.
After the initial direction of the particle is determined, the magnitude of its momentum can be gained from the curvature in the magnetic field. For charged particle in magnetic field

$$-zevB\hat{e}_r = \frac{d\vec{p}}{dt}$$

which leads to (using standard circular geometry)

$$r = \frac{p}{qB}$$

where r is the radius of the trajectory of the particle.
Now, consider three equally spaced detectors with distance $\frac{L}{2}$ between each of them. The particle swipes three points in these detectors, which form vertices of an approximately isoscales triangle (for a fast particle that does not curve much). The geometry is similar to the one displayed in figure below.
The distance r can be obtained from the Pythagoras theorem

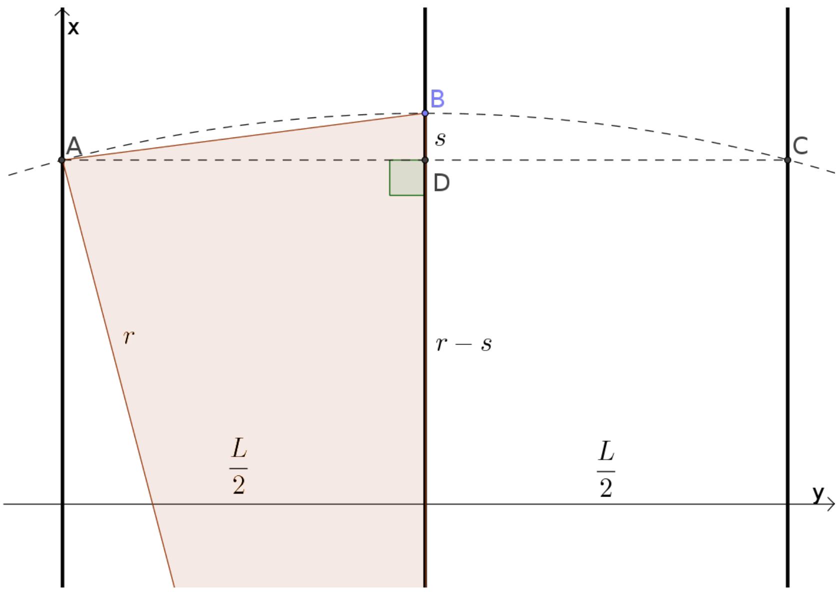
$$r^2 = \frac{L^2}{4} + (r - s)^2$$

where s is also called the sagita. Hence

$$r^2 = \frac{L^2}{4} + r^2 - 2rs + s^2$$

$$2rs = \frac{L^2}{4} + s^2$$

$$r = \frac{L^2}{8s} + \frac{s}{2}$$



Since s is usually quite small, we can write

$$r \approx \frac{L^2}{8s}$$

and hence

$$p = \frac{L^2 qB}{8s}$$

The dominant error here will be the error in determination of the sagita, which is approximately the error in determination of position in the detectors Δx. So, $\Delta s \approx \Delta x$. The error in momentum determination is then

$$\Delta p \approx \left|\frac{\partial p}{\partial s}\right| = \frac{L^2 qB}{8s^2}\Delta s \approx \frac{p}{s}\Delta x$$

We can inversly express sagita to gain error that is only dependent on p, so

$$s = \frac{L^2 qB}{8p}$$

and

$$\Delta p = \frac{p}{L^2 qB} 8p\Delta x = \frac{8p^2}{L^2 qB}\Delta x$$

and the relative error in momentum

$$\frac{\Delta p}{p} = \frac{8p}{L^2 qB}\Delta x$$

This means that larger detectors are much better at determination of the momentum than smaller detectors. Generally, anyway that we can measure more curvature yields higher precision in momentum.
In real detectors, multiple layers similar to this one are used and the agregated result is used, typically with precission about 7 % (ATLAS), achieved from individual measurements typically with precision 65 % (very bad).

4.5 Calorimetry

Calorimeters try to slow all particles to level when they start to ionize matter. Then, the calorimeters try to measure the total ionization and hence the total energy of the particle incident on the calorimeter.
Therefore, calorimeters need to absorb all secondary particles created by the particle, and therefore need to be several radiation lengths/hadronic interaction lengths long. Usually, this requirement makes calorimeters the biggest part of particle detector.
As the radiation length is approximately tenth of the hadronic interaction length, calorimeters are usually split to EM calorimeter, which is rather thin and only measures energies of electrons/positrons and photons (and π^0 particles), and hadronic calorimeter, which is bigger and measures energies of hadron interacting particles. Usually, the EM calorimeter is much more precise, because the hadronic calorimeter suffers from the undermeasurement of energy by caused by escaping non-ionizing neutrons.

Usually, the relative error in the energy determined for the particle is

$$\left(\frac{\sigma_E}{E}\right)^2 = \left(\frac{a}{\sqrt{E}}\right)^2 + \left(\frac{b}{E}\right)^2 + c^2$$

where $\frac{a}{\sqrt{E}}$ term is due to the Poissonian distribution of the generated electrons (electron ionization is approximately random Poissonian process and number of generated electrons is roughly proportional to the energy of the incident particle E) and is usually called the sampling term. Term $\frac{b}{E}$ is due to noise generated by extra electrons, which becomes smaller as the particle becomes more energetic, and c is the constant calibration term.
Tipical values for a, b and c would be $a = 0.028\sqrt{\frac{\text{GeV}}{c^2}}$, $b = 0.12\,\frac{\text{GeV}}{c^2}$ and $c = 0.003$.
Between common materials to use for calorimetry are dense materials that have small radiation and hadronic interaction lengths, such as iron (for particular detector thickness $L_R \approx 1.8$cm, $L_I \approx 17$cm), copper (similar to iron), polyethylene ($L_R \approx 50$ cm, $L_I \approx 88$ cm) or lead tungstate, $PbWO_4$ ($L_R \approx 0.9$ cm, $L_I \approx 20$ cm). Generally, L_I decreases as number of nuclei increases, while L_R decreases as number of electrons decreases, which means that

$$L_I \propto \frac{1}{density}$$

$$L_R \propto \frac{1}{Z \times density}$$

Usually, we need at least 10 lengths to have some sort of consistent output. This is usually what is used for hadronic calorimeters, as their readings are not very precise anyway. However, EM calorimeters have usually many more radiation lengths, commonly about 25 L_R, which allows for reasonable precision and capture of all EM showers, while still smaller than the hadronic calorimeter.
The sampling term of the calorimeter depends largerly on the material used, for example lead tungstate generates very small response, and thus have higher a.

4.5.1 Scintillation

When the particles ionize the material of the calorimeter, the ionization have to be somehow measured. Because calorimeters are so big and try to be very homogeneous, we cannot do the same design as in wire chamber. So, calorimeters are usually made from material which scintillates.
The ionized electron in the material is effectively in some very high orbital of its original atom. As it comes back to its original level, it emits light. If the material is transparent to this light, the light travels to the boundary of the material in random direction, where it is then read out.
Again, the amount of light measured is proportional to the ionization, so we have our information about the energy of the particle, although the calorimeter needs to be calibrated.
It should be noted that only the light at frequencies that are transparent to the material emerges. The relation to the original energy can be estimated from the materials science.
Also, as the light is emmited in random direction, there is no information about the momentum of the particle in the calorimeter.
To boost the output of the calorimeter, we can use internal reflection geometry so that we can read only one side of the calorimeter and still receive response from the majority of the calorimeter.
Anyway, calorimeters always need to be calibrated, and thus always have the c part of the error, which limits them at high energies.
The materials of the scintillators can be potentially different from the absorbers of the calorimeter, and then we need some layered structure of absorber and scintillator. Polyethylene (doped) is a classic cheap scintillator, which has good response but is not very dense, which makes it a typical choice for layered sampling calorimeter. Crystals, such as sodium iodide, have very strong response (as much as 80 000 photons for 1 MeV of ionization energy), but have slow readout (250 ns per cycle). Lead tungstate has very small response (200 photons per MeV), but are so fast that the limitation of 40 MHz of the electronics is higher than that of the readout time.

4.5.2 Sampling Calorimeters

Sampling calorimeters try to vary the absorbing part of the calorimeter with scintillating part of the calorimeter, with every scintillating part being read out separately. The advantage is that we have some sort of spatial information about the energy deposit in the calorimeter. Sometimes, even small wire chamber layer can be included for some tracking information. This than enables for example differentiation between primary and secondary electrons. The disadvantage is of course that the layered structure is inhomogeneous, and the calibration error of the calorimeter is thus increased.

5 Accelerators

The cross-section of reaction in which a massive particle appears is much higher if the energy of the particles is at least close to the rest mass of the particle occuring. Therefore, if we want to create some particles, we generally need a way how to increase the energy of particles beyond the natural limit of about 7 MeV of α particles from radioactive decays - we need to use particle accelerators.
The only viable way how to accelerate the particles is using electrical voltage, and thus only charged particles can be accelerated. Furthemore, from the form of Lorentz force $F = q(\vec{E} + \vec{v} \times \vec{B})$ it is clear that the magnetic field cannot increase the energy of the particle. This means that we need big electric fields/potential differences to accelerate the particles.

5.1 Linear Accelerators

Linear accelerators use acceleration of charged particles along a line. First type of such accelerator could use a simple static electric field. This type of accelerator is usually limited by dielectric brakedown, which limits it to maximum potential difference about 5 MV. This voltage can be achived in multiple ways. Slightly smaller but still big voltage can be achieved with Van der Graaf generator, and said 5 MV can be achieved with Cockroft-Walton diode ladder, which is a repeating circuit of diodes and capacitors which runs on alternated current and only allows the charging of capacitors. The top limit of this is dependent on the diode quality, but upper limit is the dielectric breakdown.

5.1.1 Folded Tandem Accelerator

Folded tandem accelerator starts by accelerating H^- atom across the maximum voltage towards the positively charged anode. When the atom gets close to the anode, the electrons tend to strip away from the atom, leaving only the proton, which is repelled from the anode, and thus travels the voltage again in the opposite direction. Therefore, the potential difference travelled is effectively doubled.

5.1.2 Voltage Flippers

To avoid the dielectric breakdown, alternating voltage can be used. The design is then such that tubes of metal are connected to an alternating source, with neighbouring tubes connected to the opposite voltage sign. Then, if the electron travels through tubes at the same frequency at which the voltage changes, the electron is continuously accelerated.
For the oscillating field, dielectric breakdown is smaller problem, as electrons have not enough time to properly ionize and accelerate away from the atoms.
This design can be further imporoved by creating resonant cavities that capture the electromagnetic radiation created by the oscillating fields and thus create extra electric field that can add to the acceleration of the particle.
The cavity radius must satisfy $r = \frac{2.4c}{\omega}$.
However, because the electron is accelerated, the length of the tubes have to change along the accelerator. This is generally a problem, but for highly relativistic particles, the speed does not change much even when the energy of the particle keeps increasing. Therefore, if particle is light, such as electron, it can quickly reach the relativistic regime and the tubes can be identical.
The change in speed per change in energy can be calculated as

$$\frac{dv}{dE} = \frac{1}{\frac{dE}{dv}} = \frac{1}{\frac{d(\gamma mc)}{dv}} = \frac{1}{mc\beta\gamma^3}$$

Hence for relativistic particles, $\gamma \to \infty$ and $\frac{dv}{dE} \to 0$.
The synchronisation of all cavities is critical for proper functionality of this accelerator. Otherwise, they are limited only by the length.
Usually, voltage flippers can create potential difference about 30 MV per meter of their length.
Since the particles only accelerate in certain phase, if they fall out of phase, they are decelerated until again in phase. This means that all particles are kept in a bunch. What can happen is that too many particles are pushed into a single bunch and their repulsion causes the bunch to spread - this is called the beam blow up.

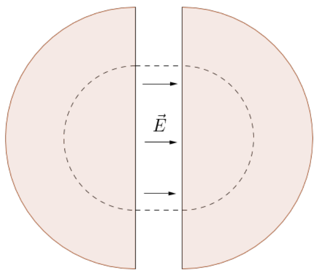
To control transverse spread of the beam of particles, magnetic lenses can be easily used.

5.2 Circular Accelerators

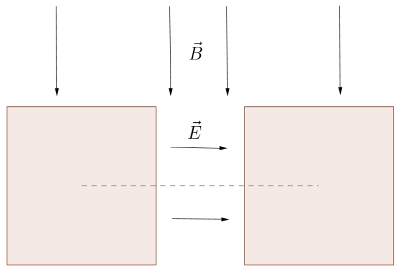
The problem with the size of the accelerator can be partially solved if we create a circular track for the particles, so that they can circulate through the potential difference several times. There are essentially two geometries for circular accelerators - cyclotrons and synchrotrons.

5.2.1 Cyclotron

Cyclotrons consists usually of two half-cylindrical shells with a gap in bewteen them (see figure below, where the particle trajectory is the dashed line)



(a) Top view



(b) Side view

The voltage on the cylindrical conducting shells changes periodically so that the particles are always accelerated by the electrical field.
The radius of the orbit is

$$r = \frac{p}{qB} = \frac{\gamma\beta mc}{qB}$$

The period of the orbit is then

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi\gamma m}{qB}$$

For small speeds, $\gamma \to 1$ and

$$T \to \frac{2\pi m}{qB}$$

which is independent of the particle speed and radius - the orbits are isochronous.
This can be generally achived if the B field is not homogeneous, but rather a function of radius so to follow $B = \gamma B_0$. From the original expression

$$r = \gamma\sqrt{1 - \frac{1}{\gamma^2}}\frac{mc}{qB}$$

$$\sqrt{\gamma^2 - 1} = \frac{qBr}{mc}$$

$$\gamma^2 - 1 = \frac{(qBr)^2}{(mc)^2}$$

$$\gamma^2 = \frac{(mc)^2 + (qBr)^2}{(mc)^2}$$

$$\gamma = \frac{qBr}{mc}\sqrt{\frac{m^2c^2}{q^2B^2r^2} + 1}$$

Assume that the cyclotron would aim to accelerate the particles to the speed of light. It would start from classical regime with condition

$$r_f = \frac{mc}{qB_0}$$

where r_f would be the radius of the orbit when the particle reaches speed of light, if it moved classically.

where B_0 is the magnetic field near the centre. Hence

$$\frac{mc}{q} = r_f B_0$$

and the previous relation for γ is

$$\gamma = \frac{Br}{B_0 r_f}\sqrt{1+\left(\frac{B_0 r_f}{Br}\right)^2}$$

Hence

$$B = \gamma B_0 = B\frac{r}{r_f}\sqrt{1+\left(\frac{B_0 r_f}{Br}\right)^2}$$

$$1 = \frac{r^2}{r_f^2}\left(1+\left(\frac{B_0 r_f}{Br}\right)^2\right)$$

$$\frac{r_f^2}{r^2} - 1 = \left(\frac{B_0 r_f}{Br}\right)^2$$

$$\frac{1}{B_0^2 r^2} - \frac{1}{r_f^2 B_0^2} = \frac{1}{B^2 r^2}$$

$$B^2 r^2 = \frac{B_0^2}{\frac{r_f^2 - r^2}{r_f^2 r^2}} = \frac{B_0^2 r_f^2 r^2}{r_f^2 - r^2}$$

Hence

$$B = \frac{B_0}{\sqrt{1-\frac{r^2}{r_f^2}}}$$

which means that the magnetic field would need to diverge as closing to r_f, which puts a limitation on the energy of the accelerated particle.
The top energy of the accelerated particle is

$$E_{max} = \gamma_{max} mc^2$$

where γ can be obtained by backwards substitution into its definition. The ratio

$$\frac{Br}{B_0 r_f} = \frac{r}{r_f}\frac{1}{\sqrt{1-\frac{r^2}{r_f^2}}} = \frac{1}{\sqrt{\frac{r_f^2}{r^2}-1}}$$

Hence

$$\gamma = \frac{Br}{B_0 r_f}\sqrt{1+\left(\frac{B_0 r_f}{Br}\right)^2} = \frac{1}{\sqrt{\frac{r_f^2}{r^2}-1}}\frac{r_f}{r} = \frac{1}{\sqrt{1-\frac{r^2}{r_f^2}}}$$

which was expected from the form of B, so everything is consistent. Hence

$$E_{max} = \frac{mc^2}{\sqrt{1-\frac{r_{max}^2}{r_f^2}}} = \frac{mc^2}{\sqrt{1-\frac{r_{max}^2 B_0^2 q^2}{m^2 c^2}}}$$

Hence we see that by increasing either r_{max} or B_0, we can achieve higher maximum energy E_{max}.
Other question is about the transverse stability of the beam of particles. The B field so far was only the z component of the field. From the symmetry, there is no ϕ component, but there could be an r component of the field.
The force acting in the z direction on the charged particle is (inside the shell) (speed for positive B_z and stable orbit must be $\vec{v} = -v\hat{e}_\phi$)

$$F_z = q(\vec{v}\times\vec{B})_z = -qv(\hat{e}_\phi \times (B_z\hat{k} + B_r\hat{e}_r))_z = qvB_r$$

This can be approximated for slowly changing radial component as

$$F_z \approx qv\frac{\partial B_r}{\partial z}z$$

Using 4th Maxwell equation for static field without any source currents (current due to orbiting particles is deemed negligable)

$$\nabla \times \vec{B} = 0$$

Hence, for the ϕ component

$$(\nabla \times \vec{B})_\phi = \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0$$

$$\frac{\partial B_z}{\partial r} = \frac{\partial B_r}{\partial z}$$

Hence we have

$$F_z \approx qv\frac{\partial B_z}{\partial r}z$$

Using Newton's third law, we have

$$\frac{d^2 z}{dt^2} = \frac{qv}{m}\frac{\partial B_z}{\partial r}z$$

This is SHM for positive charge particles (such as protons), and exponential divergence for negative charge particles (such as electrons), if the field is decreasing with radius, which it naturally is (for field between two magnets).
Hence, the angular frequency of oscillations in the z direction is

$$\omega_z^2 = \frac{qv}{m}\left|\frac{\partial B_z}{\partial r}\right|$$

For negative $\frac{\partial B_z}{\partial r}$,

$$\omega_z^2 = -\frac{qv}{m}\frac{\partial B_z}{\partial r}$$

The angular frequency of the orbit is

$$\omega_0 = \frac{2\pi}{T} = \frac{qB_0}{m}$$

Hence (since $B_0 = B_z$)

$$\nu_z^2 = \frac{\omega_z^2}{\omega_0^2} = -\frac{mv}{qB_0^2}\frac{\partial B_0}{\partial r}$$

This is the stability condition, which is satisfied if the frequency of the oscillations in the z direction is much higher than that of orbit of particles, hence if this is much higher than one. But, for increasing B_0, which we need of relativity, this is always negative, and hence the beam will always escape in the z direction. Therefore, cyclotrons have limit of about $\gamma \approx 1.5$, and afterwards the competing radial and transverse spreading cannot be both satisfied.

5.2.2 Synchrotron

Synchrotrons are the modern standard for very high energy accelerator. The advantage of such high energies is that the beams no longer tend to explode, as the distance between the protons in their rest frame is increased by a factor of γ to very big distances, so they do not interact as much.
Synchrotrons are circular tubes that use changing electric and magnetic field to accelerate the particles. The main disadvantages are the size (they need to be quite big to work with achievable B fields) and the fact that they cannot accelerate particles from slow state - some form of preacceleration is needed. Also, as particles have only 1 radius of orbit, the injections have to be timed carefully.
Furthemore, the synchrotron radiation (which goes as γ^4) prevents electrons from being accelerated at the synchrotron, as they reach terminal velocity by radiating due to acceleration in the circle. Therefore, only protons or heavier particles are used.

5.3 Experimental Accelerators

There are several experimental designs on other types of accelerators. There is a debate about building a electron/positron linear accelerator that could achieve 1 TeV, but it would require track about 30 km long if conventional cavities are used, which is very expensive. But, there is pure energy conversion by anihilation, which is nice experimentally.
There is also debate about using muon/anti-muon pair in the synchrotron, which should also completly annihilate and could reach high enough energies, but are very unstable, which is a problem (but not completly unrealistic, proton/anti-proton collider in CERN was functional and very successful).

One of the more promising designs is to use forced plasma oscillations that momentarily create extreme electric fields. The idea is to send a bunch of kicker particles that create an oscillation in plasma, which then accelerates the second bunch of particles. This was done experimentally and achieved effective voltage difference of about 50 GV/m, which is much more than the limit 100 MV/m of the conventional linear accelerators. This might be very useful in the future.

6 Colliders

6.1 Luminosity

In particle physics, the probability of certain reaction is given in the form of the cross-section of the reaction, which is labeled as σ. If there is N_2 stationary particles in a volume with cross-sectional area A in the direction of movement of one incident particle, the effective area taken by the stationary particles is

$$A_{eff} = N_2\sigma$$

Hence, if the particle lands randomly on area A, the probability of hitting some particle is

$$p = \frac{A_{eff}}{A} = \frac{N_2\sigma}{A}$$

If there is a total of N_1 particles incident, the expected number of reactions is

$$N = \frac{N_1 N_2 \sigma}{A}$$

Here, we define the luminosity as $L = \frac{N_1 N_2}{A}$, which is some form of measure of how many particles are incident on each other. This form is for luminosity when each of particles N_1 has the same probability of hitting any place in area A, and all particles N_2 are uniformly distributed in this area. In practice, we usually have Gaussian packets, for which

$$L = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y}$$

where $\sigma_{x/y}$ are the standard deviations in corresponding directions of the bunch of the particles. So, we have

$$N = L\sigma$$

Hence the reaction rate is

$$R = fN = fL\sigma = \mathscr{L}\sigma$$

where $\mathscr{L}$ is also called the luminosity, but is meant per unit time, and f is the frequency of the collider. Hence, the total number of reactions per some time T is

$$N_T = \int_0^T R dt = \int_0^T \mathscr{L}\sigma dt$$

We can see that the units of L are the inverse of σ, hence it is m^{-2}. In particle physics, these are usually recalculated to barns, with $1\ \mathrm{b} = 10^{-28}\ \mathrm{m}^2$, or femtobarns, with $1\ \mathrm{fb} = 10^{-43}\ \mathrm{m}^2 = 10^{-39}\ \mathrm{cm}^2$. In LHC, $\mathscr{L} \approx 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1} = 10\ \mathrm{nb}^{-1}\mathrm{s}^{-1}$

6.2 Particle Statistics

Usually, particle physics events are relatively rare events, which means that they obey Poissonian statistics. Therefore, the error on the number of events is usually the square root of number of mean observed number of events.
The Poisson distribution for large numbers can be well approximated as Gaussian distribution, with $\sigma = \sqrt{N}$, in accordance with the Poisson standard deviation.
The typical calculation would involve calculating the cross-section of a reaction given the integrated luminosity $\int_0^T \mathscr{L}dt$, the efficiency of the detectors (how many collisions were detected) ϵ, number of background events when the reaction did not occur N_{back} and total number of reactions N as

$$\sigma = \frac{N - N_{back}}{\epsilon \int_0^T \mathscr{L}dt}$$

The error can be found using standard procedures, and taking error on N as $\sqrt{N}$.

6.2.1 Signal Recognition

To determine the numbers N and N_{back}, we need to separate which events (sorted by some measures as product energy etc.) constitute background and which constitute the sample. This can be done by choosing an approximate area for signal and background, and calculating local event densities ρ_s and ρ_b for signal and background. We then search for points in event space where $\frac{\rho_s}{\rho_b}$ is bigger than some chosen treshold. Usually, ration $1 + \frac{\rho_s}{\rho_b}$ is rather chosen as the measure, and it is called a likelihood ratio, and has some other interesting properties, which are not discussed here.

6.2.2 Local P-Value

Local P-value can be calculated as probability that given signal occured or any other less probable signal occured. If signal is gaussian number, then the P-value for signal N would be

$$P(N, x_0, \sigma) = \int_{-\infty}^{x_0-N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} dx + \int_{x_0+N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} dx = 1 - \int_{x_0-N}^{x_0+N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-x_0)^2}{2\sigma^2}} dx$$

From the gaussian, we also define the probabilites in terms of gaussian standard deviations - σ. P-value of 5 σ (which is by convention in particle physics the minimum to declare a discovery) then corresponds to

$$P(5\sigma, 0, \sigma) \approx 2.9 \times 10^{-7} \approx 3 \times 10^{-5}\%$$

7 Final Remarks

7.1 Magnetic Field

The tracking part of a particle detector must be placed inside a magnetic field. This can be done either by surrounding just the tracker with magnets, which then requires the investigated particles to go through these magnets, causing errors in subsequent calorimetry. Other possibility is to put strong magnets around the whole detector, including the calorimeters (possibly not including the muon systems, as muons easily go through most matter). This is much more expensive.

For the geometries of magnets themselves, there are two basic setups - solenoidal and toroidal. Solenoidal geometry uses classic coil around the collider tube, which creates magnetic field parallel to the direction of motion of the colliding particles. The momentum of the product particles is then easy to measure in the transverse direction. Solenoid itself however produces so called return field on the outside. This makes it problematic when the solenoid is used inside the detector, as this return field can interfere with outer components of the detector.

The toroidal geometry consists of a coil circled into a torus, with axis of the torus coinciding with the axis of the collider. The momentum of the particles here is measured in the direction of movement, but since the field is not homogeneous and linear, the momentum determination is quite hard. Toroidal geometry is used usually inside the detector, as it does not obscure other elements of the detector.

7.2 Cyclic collisions

Other reason why cyclic colliders are currently dominating is that they can collide the same bunches several times after they reach desired speed, using therefore much more particles from each accelerated bunch. The linear accelerators cannot do that, and thus must invest more into the magnetic lenses and focusing the bunches, so to reduce area A of the bunch and increase the luminosity per one collision.