

# PX154 Physics Foundations

## Part A - Dimensional Analysis

- Dimensional Analysis can be used to determine the units of a physical quantity using its relationships to other quantities

### 1.1 Dimensions

- Dimensions are used to represent quantities rather than units as the units of a quantity may vary depending on external factors
  - Length has dimension  $L$  but can have units  $m, cm, km, ft, in$  etc

Quantity	Dimension	Units
Length	$L$	$m$
Mass	$M$	$kg$
Time	$T$	$s$
Current	$I$	$A$
Temperature	$\theta$	$K$
Amount	$mol, n$	moles
Luminous Intensity	$C, J$	$Cd$

- All terms in equations must have the same dimensions
- Angle has no dimension
- Trig functions have no dimension
- Logarithms and exponentials have no dimension

Example: Find a relationship between the time period  $T$  of a Simple pendulum and length  $l$ , mass  $m$  and acceleration due to gravity,  $g$

- $T$  can be written as a function of the other quantities

$$T = f(l, m, g)$$

- As both sides must have the same dimensions, they can be written as

$$\Pi = T^\alpha l^\beta m^\gamma g^\delta \quad \text{where } \Pi \text{ is a dimensionless constant}$$

$$[\Pi] = [T]^\alpha [l]^\beta [m]^\gamma [g]^\delta \quad \rightarrow [\ ] \text{ means 'dimension of'}$$

$$[\Pi] = T^\alpha L^\beta M^\gamma (LT^{-2})^\delta$$

$$[\Pi] = T^\alpha L^\beta M^\gamma L^\delta T^{-2\delta}$$

$$[\Pi] = T^{\alpha-2\delta} L^{\beta+\delta} M^\gamma \quad \rightarrow \text{The exponents must} = 0 \text{ as } \Pi \text{ has no dimension}$$

$$\alpha - 2\delta = 0 \quad \rightarrow \text{Taking } \alpha = 1:$$

$$\beta + \delta = 0 \quad \beta = -1/2$$

$$\gamma = 0 \quad \delta = 1/2$$

$$\gamma = 0$$

$$\therefore \Pi = T l^{-1/2} g^{1/2} m^0$$

$$\hookrightarrow T = \Pi \sqrt{\frac{l}{g}}$$

## 1.2 Buckingham Pi Theorem

- If there are several variables, they may form multiple dimensionless groups

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \dots)$$

- To determine the number of dimensionless groups,  $\tilde{N}$ , Buckingham's Pi theorem can be used

$$\check{N} = N - m$$

$N$  = number of variables

$m$  = number of fundamental independent quantities

- > Fundamental independent quantities - Quantities which, if their dimensions are raised to any powers, cannot be equated
- Mass,  $M$  and length  $L$  are fundamentally independent
  - Length  $L$  and volume  $L^3$  are not fundamentally independent

Example: The Simple pendulum

Quantity	Dimension	
$T$	$T$	$\rightarrow T$ is anticipated to count on the other variables So is not independent
$l$	$L$	$\rightarrow N=4, m=3 \quad \check{N}=4-3=1$
$g$	$LT^{-2}$	
$m$	$M$	

### 1.3 The Procedure

1. List all physical variables and their dimensions
2. Use Buckingham's pi formula to determine the number of independent dimensionless groups,  $\check{N} = N - m$
3. Form a set of groups, assuming arbitrary exponents. Solve for the exponents by requiring each product group to be dimensionless

# PX154 Physics Foundations

## Part B - Thermal Physics

### Section 1 - Heat and Temperature

#### 1.1 Temperature

> Temperature - A physical property related to the kinetic energy of the atoms or molecules of the system

- The kelvin temperature scale is the absolute temperature scale, with the lowest value  $0\text{K} = -273.15\text{K}$

$$T_{\text{K}} = T_{\text{C}} + 273.15$$

#### 1.2 Thermal Expansion

- Most materials expand as their temperature rises, and different materials expand by different amounts
  - As  $T \uparrow$ , the vibrational energy of atoms increases, so mean atomic separation increases

#### Linear Expansion

$$\Delta L = \alpha L_0 \Delta T$$

$L_0$  - Initial Length

$\alpha$  - linear expansion coefficient



## Volume Expansion

$$\Delta V = \beta V_0 \Delta T$$

$V_0$  - Initial Volume

$\beta$  - Volume expansion coefficient

- ↳ These are for isotropic materials, which have the same properties in different directions
- ↳ These are only appropriate over moderate temperature changes

## 1.3 Heat Capacity

- Heat flow,  $Q$ , into (or out of) a system will increase (or decrease) its temperature

> Specific heat capacity - the quantity of heat required to raise 1kg of a substance by  $1^\circ\text{C}$

$$Q = m c \Delta T$$

$c$  = specific heat of material

> Molar heat capacity - the quantity of heat required to raise 1 mole of a substance by  $1^\circ\text{C}$

$$Q = n C \Delta T$$

$C$  = molar heat capacity of material

$n$  = number of moles

$$C = M c$$

$M$  = mass per mole

## Latent heat

> Latent heat - The heat exchanged during a phase transition without change in temperature

$$Q = \pm mL \quad L = \text{Latent heat}$$

↳ +mL if heat is put in from system (melting, evaporation)

↳ -mL if heat is removed (freezing, liquefaction)

> Latent heat of fusion - Solid → liquid phase transitions

- +Q for melting → bonds breaking → energy released
- -Q for freezing → bonds forming → energy required

> Latent heat of vaporisation - liquid → gas phase transitions

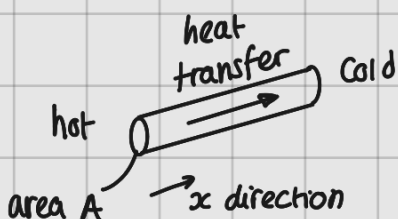
- +Q for vaporisation
- -Q for liquefaction

## 1.4 Heat transfer Mechanisms

### Conduction

- Occurs in solids and liquids
- No mass transport

Considering conduction in 1 direction:



$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

$\frac{dQ}{dt}$  - heat flow  
 $\frac{dT}{dx}$  - temperature gradient  
k - constant of thermal conductivity

↳ The minus sign is as the heat flow is from hot to cold

## Convection

- Heat flow arising from motion of a fluid

## Radiation

- A surface of temperature  $T$  will radiate a power  $P$

$$P = \frac{dQ}{dt} = e \sigma A T^4$$

$e$  - Surface emissivity

$\sigma$  - Stefan-Boltzmann constant

# PX154 Physics Foundations

## Part B - Thermal Physics

### Section 2 - Thermal Properties of Matter

#### 2.1 Ideal Gas Law

$$pV = nRT$$

$n$  = number of moles

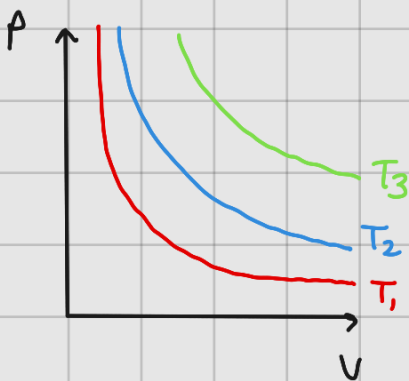
$$R = k_B N_A$$

$N$  = Number of atoms/molecules

$$pV = Nk_B T$$

↳ Works well in many different situations  
- high temperature, low pressure, low density

#### 2.2 $pV$ Diagrams



$$T_1 < T_2 < T_3$$

By holding  $n$  and  $T$  constant:

$$p \propto \frac{1}{V}$$

↳ Isotherms

#### 2.3 'Real' Gas Equation

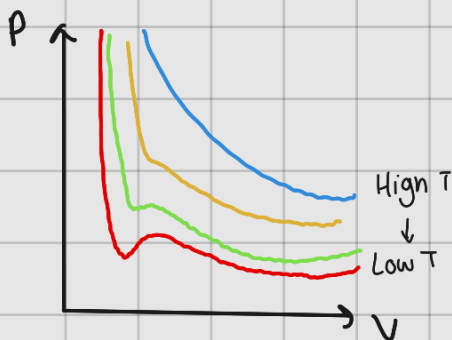
- The ideal gas equation does not account for liquefaction

A better approach is to consider:

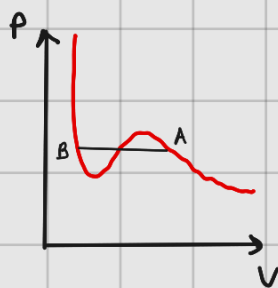
- Molecules have finite volume
  - replace  $V$  by  $V-nb$ 
    - ↳  $b$  = Co-volume - volume occupied by one mole of molecules
- Intermolecular forces
  - replace  $p$  with  $p + a \frac{n^2}{V^2}$ 
    - ↳  $a$  = constant

giving

$$\left(p + a \frac{n^2}{V^2}\right) (V - nb) = nRT$$

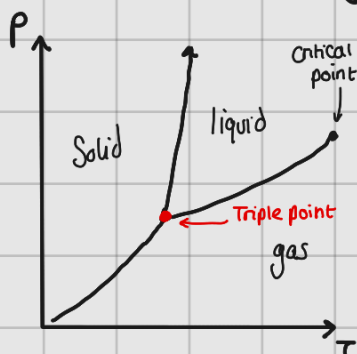


A  $pV$  diagram for the vdW equation shows a point of inflection



→ Drawing a line so areas  $x$  and  $y$  are equal, on compressing the gas, it will begin to liquefy at  $A$  until  $B$   
↳ pressure remains constant for this process

## 2.4 Phase Diagrams



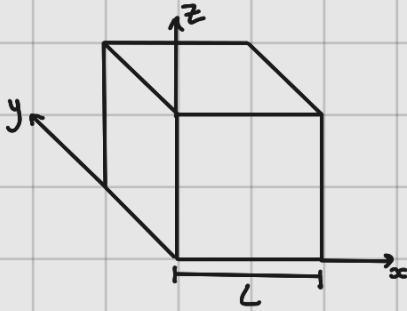
- A phase diagram shows the phase boundaries between states
- At the triple point all 3 phases coexist

- The liquid-gas boundary ends at the critical point

↳ At temperatures between this, there is no distinction between liquid and gas

## 2.5 Kinetic Molecular Model of the Ideal Gas

Consider a volume  $V$  filled with  $N$  ideal gas molecules with mass  $m$



• A molecule moving with velocity  $v$  collides normally with a wall. Its velocity is reversed giving  $\rho = 2mv_x$

• For  $N$  molecules, the number of collisions in time  $dt$  is

$$n_{\text{col}} = \frac{1}{2} \frac{N}{V} A v_x dt \quad \rightarrow \text{molecules move a distance } v_x dt \text{ in } dt$$

half of the molecules moving towards the wall at any given time

number density of molecules

The total change in momentum in time  $dt$ :

$$\begin{aligned} dQ &= n_{\text{col}} \times 2mv_x \\ &= \frac{N}{V} A m v_x^2 dt \end{aligned}$$

This causes a force  $F = \frac{dQ}{dt} = \frac{N}{V} A m v_x^2$

For an isotropic system  $v_x = v_y = v_z$  So  $v_x^2 + v_y^2 + v_z^2 = 3v_x^2 = v^2$

giving  $F = \frac{1}{3} \frac{N}{V} A m v^2$

↳ This is assuming all  $N$  molecules have the same speed  $v$ .  
Instead use the average  $\overline{v^2}$  (average of square speeds)

$$\text{Using } p = \frac{F}{A} \quad p = \frac{1}{3} \frac{N}{V} m \overline{v^2}$$

$$pV = \frac{1}{3} Nm \overline{v^2} = Nk_B T$$

This can be rearranged to give  
and  $m \overline{v^2} = 3k_B T$

$$T = \frac{m \overline{v^2}}{3k_B}$$

using  $E_k = \frac{1}{2} m v^2$  :

$$\overline{E_k} = \frac{3}{2} k_B T$$

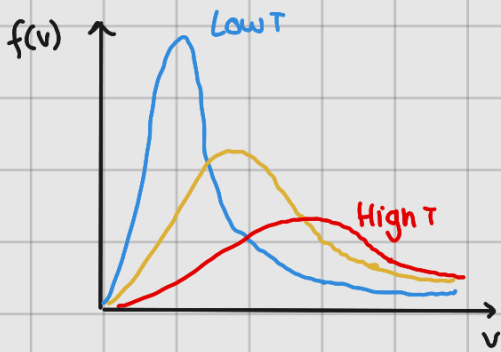
## 2.6 Maxwell-Boltzmann Distribution

The Maxwell-Boltzmann distribution of speeds is given by

$$f(v) = 4\pi \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 \exp\left( -\frac{mv^2}{2k_B T} \right)$$

$$f(E) = \frac{2}{\sqrt{\pi}} \left( \frac{1}{k_B T} \right)^{3/2} E^{1/2} \exp\left( -\frac{E}{k_B T} \right)$$





As temperature increases:

- peak moves to higher speeds
- Distribution becomes broader and flatter

Three speeds can be defined for the distribution:

$$v_{\text{prob}} = \sqrt{\frac{2k_B T}{m}}$$

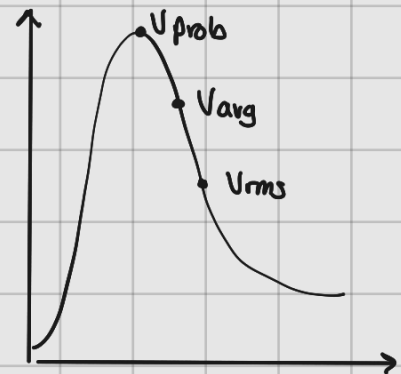
→ most probable speed

$$v_{\text{av}} = \sqrt{\frac{8k_B T}{\pi m}}$$

→ average speed

$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3k_B T}{m}}$$

→ root mean square speed



## 2.7 Collisions Between Molecules

- Molecules in a container collide with the walls of the container as well as each other

The average time between collisions is defined as

$$t_{\text{mean}} = \frac{V}{4\sqrt{2} \pi r^2 v_{\text{rms}} N}$$

$r$  = radius of spherical molecule

The mean distance or "mean free path" can then be defined as

$$\lambda = v_{\text{rms}} t_{\text{mean}} = \frac{V}{4\sqrt{2} \pi r^2 N} = \frac{k_B T}{4\sqrt{2} \pi r^2 p}$$

## 2.8 Ideal Gas Heat Capacity

>  $C_v$  - heat capacity at constant volume

>  $C_p$  - heat capacity at constant pressure

$$C_v = \frac{1}{n} \frac{dQ}{dT}$$

$\frac{dQ}{dT}$  - heat input/output

- For  $n$  moles, the translational kinetic energy  $K_{\text{tr}} = \frac{3}{2} k_B T$  can be defined as  $K_{\text{tr}} = \frac{3n}{2} RT$

- If all heat input goes into translational energy  $Q = K_{\text{tr}}$  then

$$C_v = \frac{3R}{2}$$

for monatomic

$$C_v = \frac{5R}{2}$$

for diatomic

## Equipartition of Energy

> Each form of motion is a "degree of freedom" that contributes  $\frac{1}{2} k_B T$  of energy per atom

- Translation in 3D contributes  $\frac{3}{2} k_B T$  per atom
- Rotation
  - neglected for monatomic molecules
  - diatomic can rotate around 2 axes (as 3rd is in line with atom's axis so does not contribute) so contributes  $k_B T$  per atom
- Vibration
  - kinetic and potential energy each contributes  $\frac{1}{2} k_B T$  so  $+ k_B T$  per atom
  - ↳ This does not contribute at room temperature

$$C_p = C_v + R$$

$$\gamma = \frac{C_p}{C_v}$$

### Heat Capacity of Solids

- Vibration in 3D contributes  $k_B T$  in each direction
  - ↳  $\frac{1}{2} k_B T$  for KE and PE
  - ∴  $3 k_B T$  per atom or  $3R$  per mole

$$C_v = 3R$$

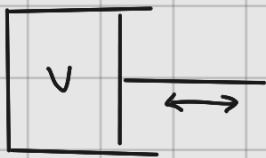
# PX154 Physics Foundations

## Part B - Thermal Physics

### Section 3 - The First Law of Thermodynamics

#### 3.1 The First Law

- The system used in this section is a closed cylindrical container with a piston that can move without friction

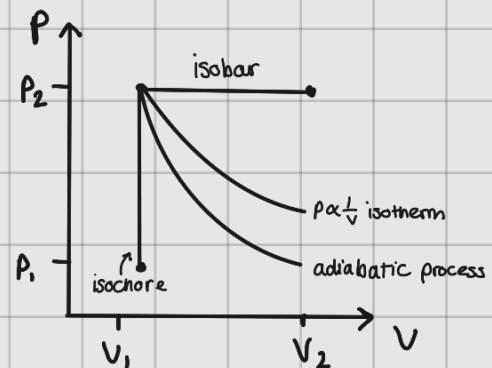


- diathermal walls
- adiabatic walls

- > diathermal - heat can be transferred perfectly through conduction
- > adiabatic - perfectly insulating, preventing heat transfer in and out

- When changing the state of a system, a 'process' is followed
- Common processes

- > isobar - pressure constant
- > isotherm - Temperature constant
- > Isochore - Volume constant
- > Adiabatic - No heat transfer



#### Clausius Statement:

- The change in the internal energy,  $\Delta U$ , of a system is equal to the difference between the heat input into the system  $Q$  and the work done by the system,  $W$

### 3.4 Work and the pV Diagram

• In the piston system, if the gas does work, it pushes the piston, increasing  $V$

- work done  $dW = F dx$

If the plunger has area  $A$ ,  $p = F/A \rightarrow dW = pA dx = p dV$

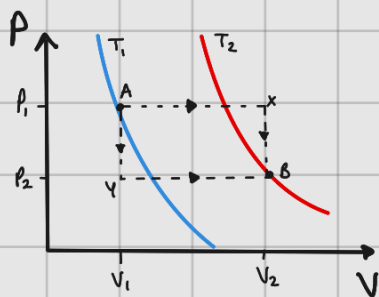
$\hookrightarrow W = \int p dV$

- For an isobar,  $p$  is constant, so  $W = p(V_2 - V_1)$

- For an isochore,  $V$  is constant, so  $W = 0$

- For an isotherm,  $W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} nRTV^{-1} dV$

### 3.5 $U, \Delta U, Q, W$



Consider 2 points A and B on a pV diagram with parameters  $p_1, V_1, T_1$  and  $p_2, V_2, T_2$  respectively

The work done and heat input/output may be different for different paths  $A \rightarrow B$

Path AXB:

$$W_{AX} = p_1 (V_2 - V_1)$$

$$W_{XB} = 0$$

Path AYB:

$$W_{AY} = 0$$

$$W_{YB} = p_2 (V_2 - V_1)$$

As  $p_2 < p_1$ ,  $W_{AYB} < W_{AXB}$

For a complete path  $AXBYA$ ,  $\Delta U$  is 0

$$W = W_{Ax} + W_{xB} + W_{By} + W_{yA}$$

$$W = p_1(V_2 - V_1) + 0 - p_2(V_2 - V_1) - 0$$

$$W = (p_1 - p_2)(V_2 - V_1)$$

### 3.6 Adiabatic Expansion of an Ideal Gas

• In an adiabatic process, there is no heat transfer, so  $Q=0$

$$dU = -dW = -pdV$$

using  $nC_v = \frac{dU}{dT}$  :  $nC_v dT = -pdV = -\frac{nRT}{V} dV$   
and  $pV = nRT$

which can be rearranged to:  $\frac{1}{T} dT = -\frac{R}{C_v} \frac{1}{V} dV$

using  $C_p = C_v + R$  and :  $\frac{R}{C_v} = \frac{C_p - C_v}{C_v} = \gamma - 1$   
 $\frac{C_p}{C_v} = \gamma$

leaving  $\frac{1}{T} dT = (1 - \gamma) \frac{1}{V} dV$

Integrating gives

$$TV^{\gamma-1} = \text{Constant}$$

It can also be found

$$pV^{\gamma} = \text{Constant}$$

for adiabatic processes

When doing work on an adiabatic process,  $W = -nC_v \Delta T$

## 3.7 Summary of Processes

Isothermal - T Constant

$$\Delta T = 0$$

$$\Delta U = 0$$

$$pV = \text{Constant}$$

$$W = \int_{V_1}^{V_2} p \, dV = nRT \ln\left(\frac{V_2}{V_1}\right)$$

Adiabatic - No heat flow

$$Q = 0$$

$$W = -nC_V \Delta T$$

$$\Delta U = -W$$

$$pV^\gamma = \text{Constant}$$

$$TV^{\gamma-1} = \text{Constant}$$

Isochore - V Constant

$$\Delta V = 0$$

$$Q = nC_V \Delta T$$

$$W = 0$$

Isobar - p Constant

$$\Delta p = 0$$

$$W = \int p \, dV = p\Delta V$$

$$\Delta T = \frac{p\Delta V}{nR}$$

$$Q = \Delta U + p\Delta V$$



# PX154 Physics Foundations

## Part B - Thermal Physics

### Section 4 - The Second Law of Thermodynamics

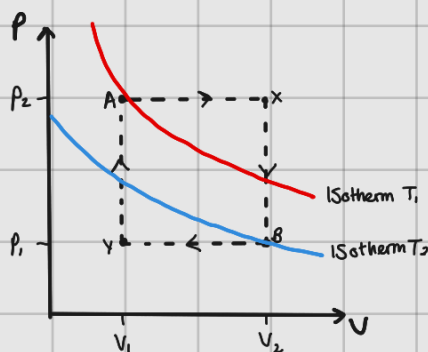
#### Kelvin-Planck Statement

It is impossible for any system to absorb heat from a reservoir at a single temperature and convert it completely into mechanical work with the system ending in the same state it began.

#### Clausius/Refrigerator Statement

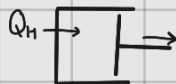
It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter object.

## 4.1 Heat Engines

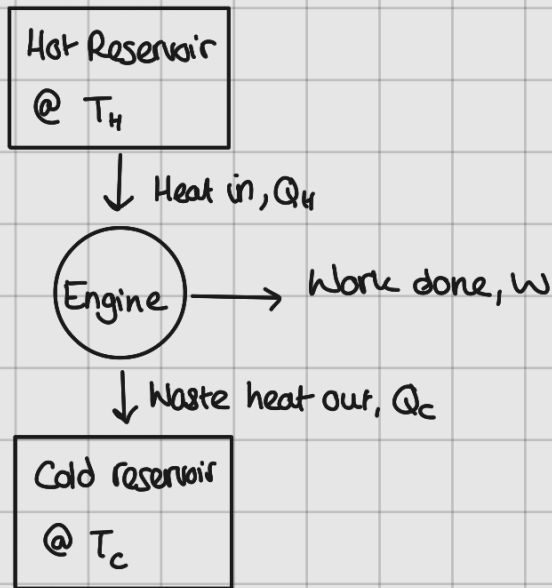


The cyclic process AXBYA can be considered on a physical system

→ Work done is area enclosed

- A → X - Isobaric expansion: Heat supplied to engine to push piston 
- X → B - Isochoric cooling: Heat engine cools to  $T_1$ .  $W=0$  and heat is expelled
- B → Y - Isobaric compression: Work is done on engine. Heat is expelled
- Y → A - Isochoric heating: Engine warms to  $T_2$ .  $W=0$  and heat must be input

This can be represented as



$$W = Q_H + Q_C$$

$$W = |Q_H| - |Q_C|$$

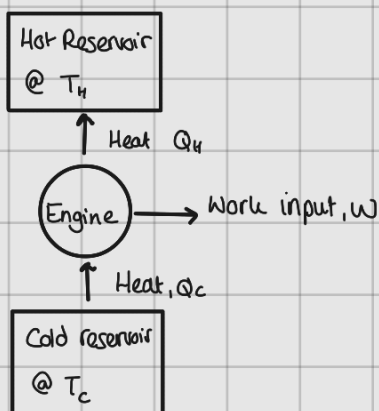
↳ When heat is leaving the engine,  $Q_C < 0$

Efficiency

$$\epsilon = \frac{W}{Q_H}$$

## The Fridge and Heat pumps

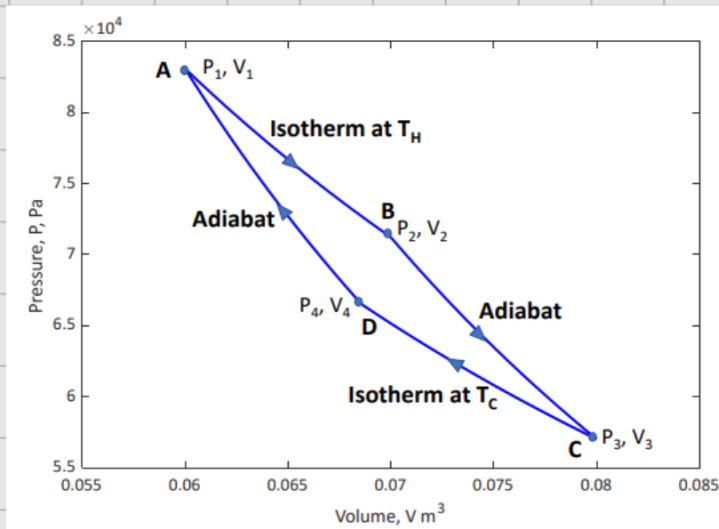
- Heat can be transferred from a cold to hot system by doing work on an engine



- Work  $W$  removes heat  $Q_C$  from CR
- Efficiency (performance coefficient)

$$k = \frac{|Q_C|}{|W|}$$

## 4.2 Carnot Cycle



↳ Ideal model of a heat engine  
- most efficient cycle

A → B: Isothermal expansion at  $T_H$

$$\Delta U = 0$$

$$Q_{AB} = W_{AB}$$

$$W_{AB} = \int_{V_1}^{V_2} nRT V^{-1} dV = nRT_H \ln(V_2/V_1)$$

B → C: Adiabatic Expansion

$$\Delta U = -W_{BC}$$

$$Q_{BC} = 0$$

$$W_{BC} = -nC_V \Delta T = -nC_V (T_C - T_H)$$

C → D: Isothermal compression at  $T_C$

$$\Delta U = 0$$

$$Q_{CD} = W_{CD}$$

$$W_{CD} = \int_{V_1}^{V_2} p dV = nRT_C \ln(V_4/V_3)$$

D → A: Adiabatic Compression

$$\Delta U = -W_{DA}$$

$$Q_{DA} = 0$$

$$W_{DA} = -nC_V (T_H - T_C)$$

Total:  $\Delta U = 0$

$$Q = Q_{AB} + Q_{CD}$$

$$= nR \left( T_H \ln\left(\frac{V_2}{V_1}\right) + T_C \ln\left(\frac{V_4}{V_3}\right) \right)$$

$$W = Q_{AB} + Q_{CD}$$

Using  $TV^{\gamma-1} = \text{constant}$  for adiabatic processes:

$$T_H V_2^{\gamma-1} = T_C V_3^{\gamma-1} \quad T_H V_1^{\gamma-1} = T_C V_4^{\gamma-1}$$

Which can be simplified to  $\frac{V_2}{V_1} = \frac{V_3}{V_4}$

hence

$$W = Q = nR(T_H - T_C) \ln(V_2/V_1)$$

Efficiency of  
Carnot Cycle

$$\epsilon = 1 - \frac{T_C}{T_H}$$

### 4.3 Reversible and Irreversible processes

- When a hot object is placed next to a cold one, heat is transferred hot  $\rightarrow$  cold until thermal equilibrium
  - > This is dynamic equilibrium - heat transferred back and forwards at same rate
  - process is irreversible - heat will not be transferred to return to initial state
- For a process to be reversible, it must be considered to be adiabatic

- The most efficient cycle is a reversible cycle
  - All real processes are irreversible

## 4.4 Entropy

- In the example of heat being transferred from a hot to cold object, the system has become "more random" and has gained entropy  $S$ 
  - If heated water is cooled, entropy decreases to its original value so for the water  $\Delta S = 0$  but entropy of the universe has likely increased
- Entropy is a function of state

$$\Delta S = \int_{\text{State 1}}^{\text{State 2}} \frac{dQ}{T}$$

eg. use  $dQ = mc dT$

# PX154 Physics Foundations

## Part C - Waves

Wavefunction:

$$U(x,t) = A \cos(kx \pm \omega t)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f$$

↳  $A$  = amplitude

↳  $+$  → left travelling wave

↳  $k$  = wavenumber

↳  $\omega$  = angular frequency

$$c = f\lambda = \frac{\omega}{k}$$

> Transverse/longitudinal wave - wave oscillations are perpendicular/parallel to direction of travel

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

In 3D,  $\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$

for EM waves:  $\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 E}{\partial x^2}$        $E$  = electric field

Phase difference introduced as  $u(x,t) = A \cos(kx - \omega t + \phi)$

$$0 \leq \phi < 2\pi$$

Superposition (Adding waves)

SAME  $f \rightarrow$  use  $\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

for  $u_1 = A_1 \cos(kx - \omega t + \phi_1)$        $u_2 = A_2 \cos(kx - \omega t + \phi_2)$

$$\Delta\phi = \phi_1 - \phi_2$$

- If  $\Delta\phi = 0$ ,  $u_{\text{tot}} = u_1 + u_2 = 2u_1 = 2u_2$

- If  $\Delta\phi = \pi$   $u_{\text{tot}} = 0 \rightarrow$  Destructive interference

SIMILAR F

$$kx - \omega t$$

$u_1$  has  $\omega + \Delta\omega$  and  $k + \Delta k \rightarrow A \cos(x(k + \Delta k) - t(\omega + \Delta\omega))$

$u_2$  has  $\omega - \Delta\omega$  and  $k - \Delta k \rightarrow A \cos(x(k - \Delta k) - t(\omega - \Delta\omega))$

$$u_1 + u_2 = A \cos(kx - \omega t) \cos(\Delta kx - \Delta\omega t)$$

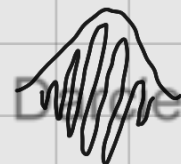
$\rightarrow$  for 2 frequencies  $f + \Delta f$  and  $f - \Delta f$  superposing:

-  $f$  is heard,  $2\Delta f$  is the beat frequency ( $f_1 - f_2$ )

transverse velocity  $v_y(x,t) = \frac{\partial u(x,t)}{\partial t}$   
of a particle

Phase velocity,  $v_p$  - the velocity of one phase of a wave

group velocity,  $v_g$  - the velocity of an 'envelope wave' - a superposition of multiple waves of varying frequencies





$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk} = v_p + k \frac{dv_p}{dk}$$

> Dispersion - the relationship  $\omega(k)$

$$c = f\lambda = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

Power transferred along a wave:

$$P(x,t) = \omega^2 A^2 \sqrt{\mu T} \sin^2(kx - \omega t)$$

$$P_{av} = \frac{1}{2} \omega^2 A^2 \sqrt{\mu T}$$

$$Z = \sqrt{\mu T} \rightarrow \text{Impedance}$$

For a longitudinal wave along a 1D rod:

$$\frac{\partial^2 \sigma}{\partial x^2} = \frac{\rho}{Y} \frac{\partial^2 \sigma}{\partial t^2}$$

$\sigma = \text{Strain}$     $Y = \text{Stress/Strain}$

$\rho = \text{density}$

↳ for transverse shear wave, use Shear modulus,  $G$

$$v_{\text{sound}} = \left( \frac{B}{\rho} \right)^{1/2}$$

$B = \text{bulk modulus} = -V \frac{dp}{dV}$     $p = \text{pressure}$   
 $V = \text{volume}$

→ in gases + liquids

> Doppler Effect - the apparent change in frequency as measured by an observer moving relative to the source

Sound Waves:

$$f' = f \left( \frac{C + U_o}{C + U_s} \right)$$

$f' > f$  if moving closer  
 $f' < f$  if moving apart

$f'$  = freq observed       $f$  = freq emitted

$U_o$  = observer Speed       $U_s$  = Source Speed

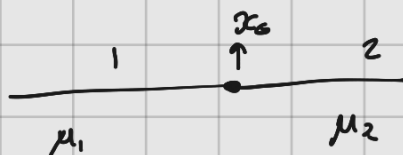
$C$  = Speed of sound

EM Waves

$$f' = f \left( \frac{C - v}{C + v} \right)^{1/2}$$

$v$  = relative Speed of source + observer

For 2 joined strings:



- T same  
 - f same

$$v = \sqrt{\frac{T}{\mu}}$$

3 waves: incident  $y_i$ , reflected  $y_r$ , transmitted  $y_t$   
 you can find the transmission and reflection coefficients

$$t = \frac{A_t}{A_i} = \frac{2Z_1}{Z_1 + Z_2}$$

$$r = \frac{A_r}{A_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$Z = \sqrt{\mu T}$  = impedance of string

Polarization: If light is passed through 2 polarizing filters, the transmitted intensity through the 2nd filter is given by

$$I_t = I_0 \cos^2 \theta$$

$\theta$  = angle between filters

Refractive index

$$n = \frac{c}{v}$$

$v$  = Speed of light in material

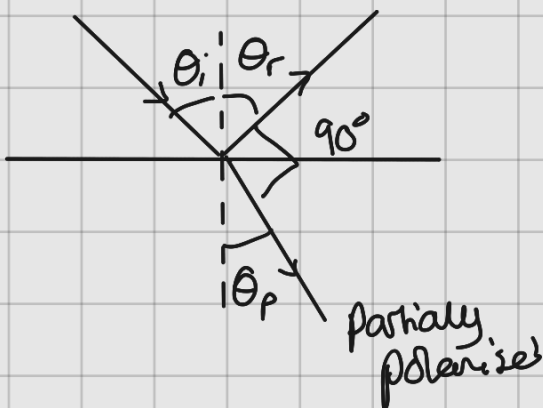
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Total internal reflection:

- if  $n_1 > n_2$

$$\sin \theta_c = \frac{n_2}{n_1}$$

• Light can be polarized by reflection



$$\tan \theta_p = \frac{n_2}{n_1}$$

Transmission and Reflection Coefficients:

↓ Give the fraction of light reflected / Transmitted

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}$$

$$T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$