PX158 - Astronomy

Section 1 - Data & Equations

Solar mass: $1M_\odot = 1.98 \times 10^{30}\,kg$

Solar radius: $1R_\odot = 6.9 \times 10^{8}\,m$ $\qquad R_{earth} \sim \frac{1}{100} R_\odot \qquad R_{jup} \sim \frac{1}{10} R_\odot$

astronomical unit = earth-sun distance: $1\,au = 1.5 \times 10^{11}\,m$

parsec: $1\,pc = 3.09 \times 10^{16}\,m$

Arc-Second: $1'' = \frac{1}{3600}$ degrees $\qquad t_0 = H_0^{-1} = 13.6\,Gyr$

Conversions

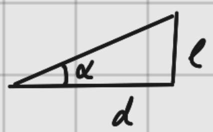
$1\,ly = c \times 3600 \times 24 \times 365\,m$ $\qquad 1\,pc = \dfrac{1\,au}{1''}$

Equations:

For $\delta < \phi$: $\quad alt_{max} = 90^\circ + \delta - \phi$

For $\delta < \phi$: $\quad alt_{max} = 90^\circ - \delta + \phi$

$\boxed{\alpha = l/d}$ for $d \gg l$ and α in rad.



$$\boxed{d = \frac{r_E}{p} = \frac{1\,AU}{p}}$$

d = distance from sun to star

p = parallax angle

$$\boxed{f = \frac{L}{4\pi d^2}}$$

d = distance to source $\qquad L$ = luminosity of object

f = flux = power per unit area

$$m_2 - m_1 = -2.5 \log_{10}\left(\frac{f_2}{f_1}\right)$$

m_n = magnitude of star n
f_n = flux of star n

$$m = -2.5 \log_{10}\left(\frac{f}{f_{vega}}\right)$$

$$m - M = 5 \log_{10}\left(\frac{d}{10pc}\right)$$

m = apparent magnitude
M = absolute magnitude

$$\text{magnifying power} = \frac{\pi S_B}{4}\left(\frac{f}{D}\right)^{-2}$$

S_B = surface brightness (flux per angle²)
f = lens focal length
D = lens/mirror diameter

$$\alpha_{min} = 1.22 \frac{\lambda}{D}$$

D = circular telescope diameter
α_{min} = angular resolution (radians)

$$\frac{\alpha_2}{\alpha_1} = \frac{f_1}{f_2}$$

f_n = focal length of lens n
α_n = angle between ⊥ axis of lens and light
α_1/α_2 = angular magnification

$$B_\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

B_λ = power per wavelength

$$\lambda_{max} T = 2.898 \times 10^{-3}\ mK$$

$$J = \sigma T^4$$

J = power per unit area

$$\boxed{L = 4\pi R^2 \sigma T^4}$$

L = Luminosity of spherical black body

$$\boxed{m_1 r_1 = m_2 r_2}$$

← For binary star system

$R = r_1 + r_2$

$$\boxed{F = \frac{G m_1 m_2}{R^2}}$$

Kepler's third law:

$$\boxed{a = \omega^2 r}$$

$$\boxed{\omega^2 = \frac{4\pi^2}{P} = \frac{G(m_1 + m_2)}{R^3}}$$

$$\boxed{\lambda = \lambda_0 \left(1 + \frac{v}{c}\cos\theta\right) = \lambda_0 \left(1 + \frac{V_R}{c}\right)}$$

V_R = radial velocity

$V_R = V\cos\theta$

$$\boxed{\Delta\alpha = \frac{4GM}{c^2 r}}$$

r = distance between light and mass

$\Delta\alpha$ = angle of deflection

$$\boxed{\theta_E = \sqrt{\frac{4GM}{Dc^2}}}$$

θ_E = Einstein Ring radius

D = Distance to object

($2D$ = distance to light source)

$$\boxed{T = \left(\frac{L_\odot}{16\pi\sigma d^2}\right)^{1/4}}$$

d = distance to Sun

T = Temperature of object in Solar System

$$\boxed{\delta = \frac{\pi R_p^2}{\pi R_s^2} = \left(\frac{R_p}{R_s}\right)^2}$$

(For an exoplanet at $i = 90°$ crossing its host star)

R = radius of planet / star

δ = depth of transit

$$D = \frac{P}{\pi} \sin^{-1}\left(\frac{R_p + R_s}{a}\right)$$

D = transit duration

a = orbital radius

$$P_c = \rho \frac{GM}{R} = \frac{3}{4\pi} \frac{GM^2}{R^4}$$

P_c = Central pressure

$$P = n_d k_B T = \frac{n N_A}{V} k_B T = \frac{\rho k_B T}{\mu}$$

n_d = number density of particles

μ = mass per unit particle

$$k_B T_c = \frac{GM\mu}{R}$$

T_c = central temperature

μ = mass per unit particle

$$t = \frac{E}{L}$$

t = lifetime of Star

E = total energy available

L = luminosity

$$t \propto \frac{1}{M^3}$$

$$E_d = \frac{p^2}{2m} = \frac{h^2 n_e^{2/3}}{2m_e}$$

E_d = degeneracy energy

n_e = electron number density

$$R_s = \frac{2GM}{c^2}$$

R_s = Schwarzchild radius

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

ρ_c = density of universe

$$M = \frac{V^2 R}{G}$$

M = mass interior to star's orbit

V = velocity of Star

R = radius of Star in galaxy

$$V = H_0 d$$

v = recession Speed

H_0 = Hubble constant

d = distance to galaxy

$$V = cz$$

v = recession velocity

Definitions

> Altitude - Angle of an object above observer's horizon

> Azimuth - Angle of object measured from North around the horizon

> Meridian - Imaginary line in sky running North → South

> obliquity - Angle between a planet's spin axis and orbital axes (Earth = 23.5)

> Sidereal day - Time taken for Earth to rotate once relative to "fixed" stars

> Solar day - Time taken for Earth to rotate relative to the Sun

> Synodic day - 'Solar day' but for any planet

> zenith - point directly above an observer

> flux - power per unit area as seen by an observer

> Absolute magnitude (M) - The magnitude of an object if it was at a distance of 10pc

> angular resolution - The smallest angle between 2 stars where one can distinguish them as 2

> Seeing - Turbulence in Earth's atmosphere blurs images of astronomical objects for ground-based telescopes

> Astrometry - The measurement of the positions, motions and magnitudes of stars

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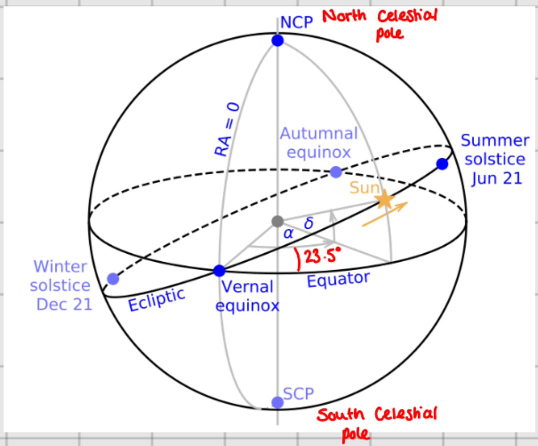
Section 2 - The Celestial Sphere

- A way of mapping the position and movement of stars and planets using a reference grid



Ra = Right ascention (vertical)
$0 \to 360°$ or $0 \to 24h$
Dec = Declination (horizontal)

- Celestial Sphere rotates once per day
- The Sun moves along the ecliptic (full circle across 1 year)
  - Ecliptic offset by 23.5°

Vernal equinox
- RA=0, Dec=0
- Sun rises due East at 6am, sets due West at 6pm
- ~ 21st March

Summer Solstice
- RA = 90 Dec = 23.5°
- Sun hits peak of ecliptic → longest day of the year
- ~ 21st June

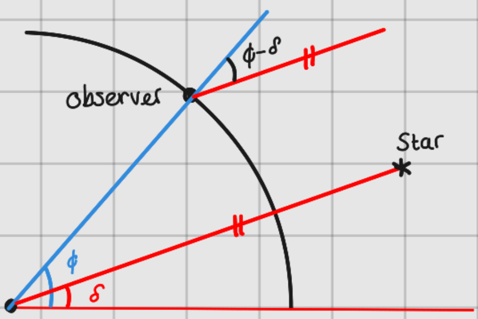
Autumn Equinox

- RA = 180   Dec = 0
- Sun rises due East and sets due West
- ~23rd September

Winter Solstice

- RA = 270°   Dec = -23.5°
- Sun reaches lowest point in ecliptic → Shortest day of year

↳ Seasons reversed for Southern hemisphere



δ = declination

ϕ = latitude of observer

> Altitude, a – Angle of an object above observers horizon

> Azimuth – Angle of an object measured from north around the horizon (N=0°, E=90°, S=180°, W=270°)

↳ horizon coordinates

> Meridian – N-S line for observer

· The star above reaching its highest point in the sky crossing the meridian

For $\delta < \phi$: $alt_{max} = 90° + \delta - \phi$

For $\delta < \phi$: $alt_{max} = 90° - \delta + \phi$

> Solar/Synodic day – time for 1 day measured by the Sun crossing the meridian

- 24h → 365 Solar days in a year

> Sidereal day – time taken for 1 day measured by Earths rotation to 'fixed' Stars
  - $23^h\ 56^m$
  - Increasing with time as Earth's Spin period Slows over time

- Samerian Calendar had 12^m of 30^d with leap month every ~4^y
- Julian calendar had 365^d + leap day every 4^y
- Gregorian calendar Skips leap years

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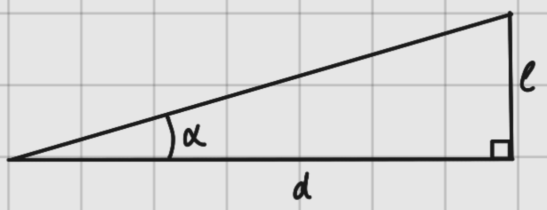
Section 3 - Angles and Parallax

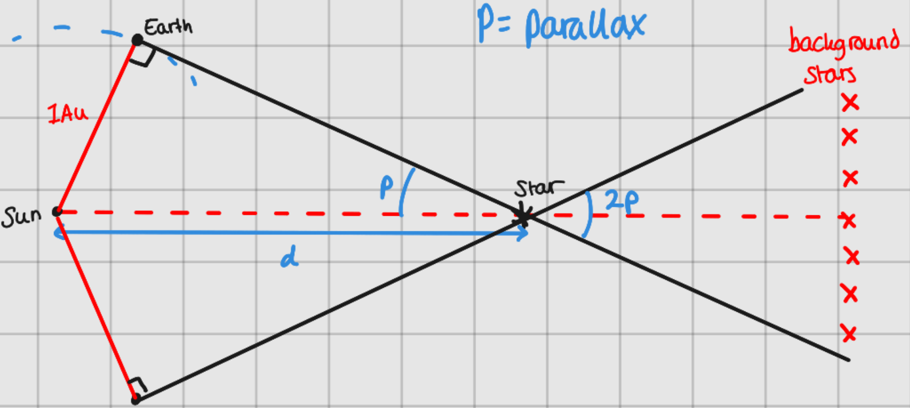
1 arcsecond $= 1'' = \frac{1}{3600}°$ $1° = 3600''$



$\tan\alpha = l/d$

For $d \gg l$: $\boxed{\alpha = l/d}$ with α in radians

Parallax



As the earth is moving around the Sun, the positions of stars move slightly each year

- A target star is observed 2 times, 6^m apart when the Earth-Sun vector is perpendicular to the target
  - If the total change in angular position is $2p$, the distance from the Sun to the star is

$$\boxed{d = \frac{r_E}{p} = \frac{1\,AU}{p}}$$

d = distance from sun to star $d \gg 1\,au$

p = parallax angle

> parsec - parallax of 1 arcsecond

$$1\,pc = \frac{1\,au}{1''} = \frac{1.5\times10^{11}\,m}{(1/3600)° \times \frac{\pi}{180}} = 3.094\times10^{16}\,m = 3.27\,ly$$

Proper Motion

> Proper motion – fixed stars show motion over many years
→ measurements need to be made over a number of years to separate proper motion and parallax

Proper motion $\equiv \mu$ measured in milli-arcseconds/year
Represented as μ_{RA} and μ_{DEC}

- To get 3D space velocity, radial velocity would need to be measured too

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Section 4 - Fluxes and Magnitudes

Flux

> Luminosity - power of an object (W)

> Flux - power per unit area as seen by an observer

$$\boxed{f = \frac{L}{4\pi d^2}}$$

d = distance to source

Magnitudes

- Stars catalogued from brightest (m=1) to dimmest (m=6)
- Due to logarithmic eye, $\uparrow \Delta 5$ mag = $\downarrow$ x100 flux

For 2 Stars:

$$\boxed{m_2 - m_1 = -2.5 \log_{10}\left(\frac{f_2}{f_1}\right)}$$

- Each Star has an 'apparent magnitude' relative to Vega ($m_v = 0$)

$$\boxed{m = -2.5 \log_{10}\left(\frac{f}{f_{vega}}\right)}$$

> Absolute magnitude (M) - The magnitude of an object if it was at a distance of 10pc

$$\boxed{m - M = 5\log_{10}\left(\frac{d}{10pc}\right)}$$

m = apparent magnitude
M = absolute magnitude

- Humans can see to +6 magnitudes with the naked eye

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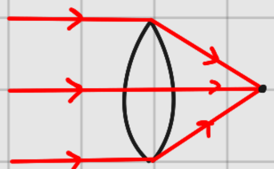
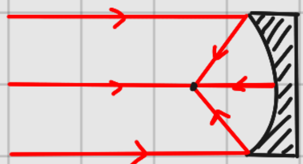
Section 5 - Telescopes

- 'Light' gathering power is proportional to area used to collect light
- Telescopes increase light gathering power, magnify light flux and angular resolution

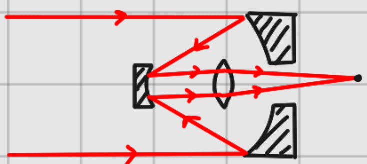
Telescopes



- Refracting telescopes use a lens to focus light
  - Simple and cheap if small



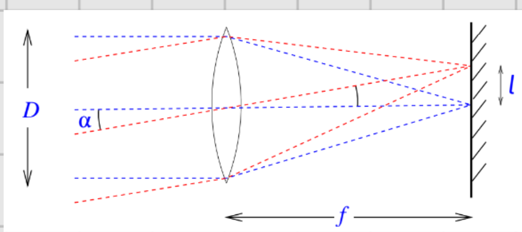
- Reflecting telescopes use a parabolic mirror



- Combination telescopes use both mirrors and lenses

Cameras

- Simple refracting telescopes can be thought of as cameras



- Rays through lens' centre pass unchanged
- Parallel incident rays meet at focal point

$l = \alpha f$ from small angle approx.

- S_B = Surface brightness - flux per angle squared
- D = diameter of telescope mirror/lens

Power into telescope $= \frac{S_B \alpha^2 \pi D^2}{4}$

Power per unit area $= \frac{S_B \alpha^2 \pi D^2}{4 \ell^2} = \frac{S_B \alpha^2 \pi D}{4 f^2 \alpha^2} = \frac{S_B \pi}{4} \underbrace{\left(\frac{f}{D}\right)}_{\text{focal ratio } D}^{-2}$

$$\boxed{\text{magnifying power} = \frac{\pi S_B}{4} \left(\frac{f}{D}\right)^{-2}}$$

- Telescopes have low focal ratios → fast optics

Angular Magnification

- Diffraction in telescopes limits angular resolution

> angular resolution - The smallest angle between 2 Stars where one can distinguish them as 2

$$\boxed{\alpha_{min} = 1.22 \frac{\lambda}{D}}$$

D = circular telescope diameter

α_{min} = angular resolution (radians)

> Seeing - Turbulence in Earth's atmosphere blurs images of astronomical objects for ground-based telescopes

↳ depends on light path through atmosphere

- "Adaptive optics" can compensate for seeing using deformable mirrors

- A telescope with an eyepiece provides angular magnification

$$\boxed{\frac{\alpha_2}{\alpha_1} = \frac{f_1}{f_2}}$$

f_n = focal length of lens n

α_n = angle between ⊥ axis of lens and light

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Section 6 - Black Bodies and Colours

Black-bodies

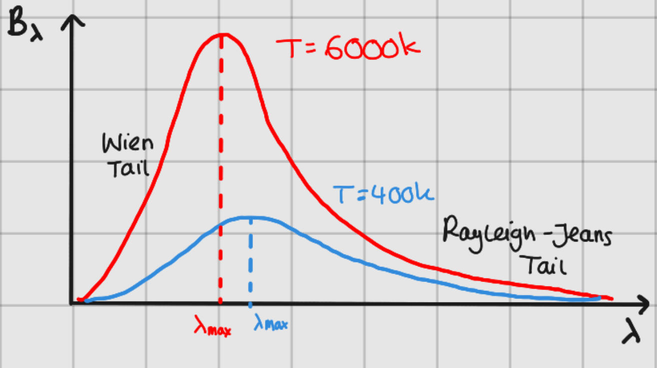
> Black body - a perfect emitter and absorber of radiation
  ↳ Absorbs all radiation incident and emits all radiation contained

Planck's equation for blackbody radiation

$$\boxed{B_\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}}$$

B_λ = power per wavelength

$B_\lambda(\lambda)\,d\lambda$ represents power per unit area emitted between wavelengths λ and $\lambda + d\lambda$



→ Steep cutoff as $\lambda \to 0$ - Wien's Tail
→ Slow decrease as $\lambda \to \infty$ - Rayleigh-Jeans Tail

• As $T\uparrow$, $B_\lambda \uparrow$ for all wavelengths

$$\boxed{\lambda_{max} T = 2.898 \times 10^{-3}\ mK}$$

$$\boxed{J = \sigma T^4}$$

J = Power per unit area

$$\boxed{L = 4\pi R^2 \sigma T^4}$$

L = Luminosity of spherical black body

Colours

- Temperatures of astronomical objects are measured by measuring flux over a range of wavelengths
  ↳ colour filters applied

- The difference in magnitude between 2 filters X and Y, m_x and m_y, is directly related to the flux ratio f_x/f_y

- Filters are described by letters UBVRI (ultraviolet, blue, visible, red, infrared)

- A star with magnitude $m_B = B = 12.34$ and $m_V = V = 11.67$ will have a colour index $B-V = +0.67$

- Vega is used as a calibration point as $B-V=0$
  - Stars with $B-V>0$ are cooler than Vega
  - Stars with $B-V<0$ are hotter than Vega

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Section 7 - Astronomical Masses

- Binary Stars provide a method for measuring mass, as $F_{12} = F_{21}$

1. $\boxed{F = \frac{G m_1 m_2}{R^2}}$

2. $\boxed{m_1 r_1 = m_2 r_2}$ $\quad R = r_1 + r_2$

3. $\boxed{a = \omega^2 r}$ $\quad \omega$ = angular velocity

By equating $F = ma = m\omega^2 r$ to $G m_1 m_2 R^{-2}$, an equation for angular velocity can be derived to be

$$\boxed{\omega^2 = \frac{4\pi^2}{P} = \frac{G(m_1+m_2)}{R^3}}$$

- "Visual binaries" are binary star systems that can be spatially resolved for both stars
  - ↳ Most binary stars are not this
- Spectroscopic binaries can be observed through 2 sets of spectra
  - Doppler Shift used to determine star velocity

- For an object with speed V at angle $\theta°$ to our line of sight:
  - $\theta = 0°$ is moving away, $\theta = 180°$ = moving towards

$$\boxed{\lambda = \lambda_0\left(1 + \frac{V}{c}\cos\theta\right) = \lambda_0\left(1 + \frac{V_R}{c}\right)}$$

V_R = radial velocity

$V_R = V\cos\theta$

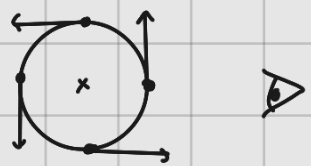
For a star in an edge-on orbit

$$\boxed{V_R = \gamma + V_{orb} \sin(i) \sin\left(\frac{2\pi}{P}(t - T_0)\right)}$$

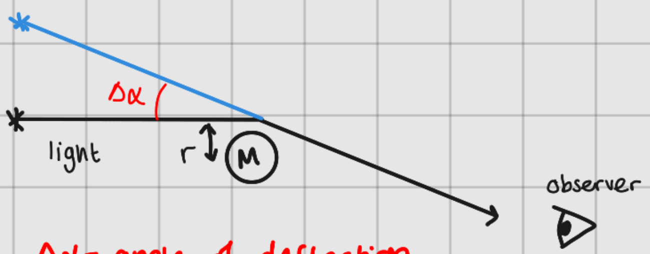
(ignoring Lorentz factor)

γ = radial velocity of CoM
T_0 = arbitrary reference time
V_{orb} = orbital speed
i = orbital inclination

- For eclipsing binaries, $i = 90°$



Gravitational lensing



Δα = angle of deflection

- General relativity predicts light will bend in a gravitational field

$$\boxed{\Delta\alpha = \frac{4GM}{c^2 r}}$$

r = distance between light and mass
$\Delta\alpha$ = angle of deflection

- The bending of light rays due to gravity mean that massive objects can act as lenses
- If a foreground massive object lines up perfectly with a light source for an observer, it will bend light
  - observer will see an 'Einstein Ring' of light surrounding the position of the source

$$\boxed{\theta_E = \sqrt{\frac{4GM}{Dc^2}}}$$

θ_E = Einstein Ring radius

D = Distance to object

(2D = distance to light source)

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Section 8 - The Solar System and Exoplanets

Our Solar System

- Sun
- 4 "Rocky" planets
- 4 Giant Planets (Ice, gas)
- Asteroid belt (rocky material)
- Kuiper belt (Icy bodies, source of short period comets, 30-50 AU)
- Oort Cloud (icy bodies, Source of long period comets, 50000 AU)

- For an object of radius R, temperature, distance d from the sun $(d \gg R)$, it will appear as a circular disc to the Sun
- Assuming it acts as a blackbody, it absorbs

$$\frac{L_\odot}{4\pi d^2}\pi R^2$$

→ from $f = \frac{L_\odot}{4\pi d^2}$ flux = power per unit area

→ Area

And radiates $4\pi R^2 \sigma T^4$ → from $L = 4\pi R^2 \sigma T^4$

If in thermal equilibrium: $4\pi R^2 \sigma T^4 = \frac{L_\odot}{4\pi d^2}\pi R^2$

$$\boxed{T = \left(\frac{L_\odot}{16\pi\sigma d^2}\right)^{1/4}}$$

d = distance to sun

Inner Planets:
- Core made from metal with Silicate mantle
- Secondary atmospheres due to volcanic activity and comet impacts

Outer planets:
- Low density, composed of hydrogen, helium
- Core of iron, nickel, Silicates
- Atmosphere's are primary → formed with the planet
- Have many moons

Comets:
- Icy bodies
- Deliver H_2O and other molecules to planets

The Sun:
- Made up of hydrogen, helium
- Most of the mass of the Solar System
- Formed with planets

Exoplanets

> Exoplanet – planet orbiting another star
↳ Very difficult to detect as they do not emit light
- First known exoplanets found from doppler Shift of their host stars

Radial velocity technique:
$m_1 r_1 = m_2 r_2$ can be used to find the radius of the planet, then $v_1 = r\omega = \frac{2\pi r}{p}$

- Planets can migrate significantly in orbital radius within their Solar System

Transit method:

- If the orbital plane of an exoplanet happens to lie close to edge on ($i = 90^\circ$) as seen from Earth, it will 'transit' across the stellar disc
  - results in apparent flux from the star periodically

The depth of transit, δ, is given by the ratio area of planet to star

$$\boxed{\delta = \frac{\pi R_p^2}{\pi R_s^2} = \left(\frac{R_p}{R_s}\right)^2}$$

R = radius of planet / star

δ = depth of transit

The duration of transit is given by

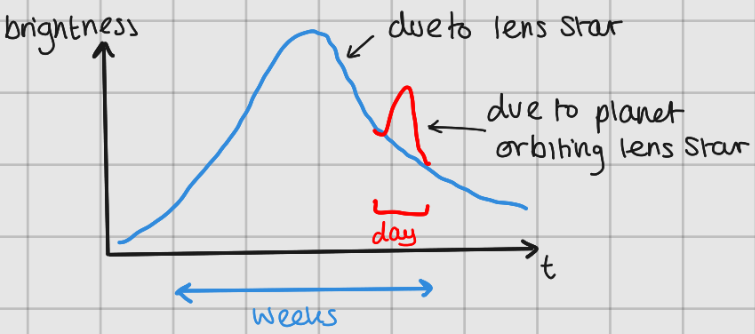
$$\boxed{D = \frac{P}{\pi} \sin^{-1}\left(\frac{R_p + R_s}{a}\right)}$$

D = transit duration

a = orbital radius

> Microlensing - gravitational lensing of stars in our galaxy

- Sometimes the lensing object can be a star and exoplanet



↳ If the lensing star has a planet with orbital radius close to Θ_E, the planet will cause further gravitational lensing

- Allows us to detect planets at 1-2AU ← very difficult otherwise

> Astrometry - The measurement of the positions, motions and magnitudes of stars

> Direct imaging - Young exoplanets have higher temperatures → more thermal emission → brighter

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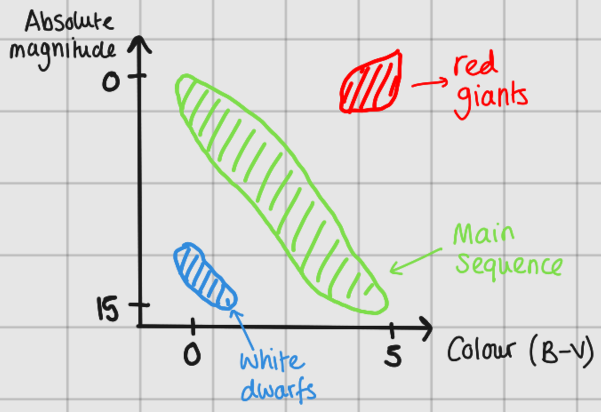
Section 9 - Stars

HR Diagrams

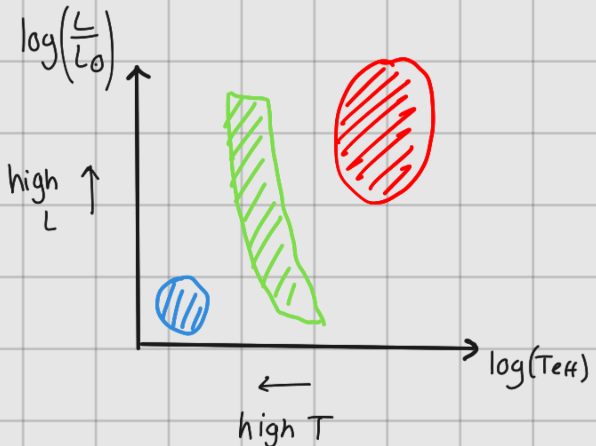
- Hertzsprung-Russel diagrams show stars lie in distinct regions of colour and absolute magnitude, and there is correlation between luminosity and temperature



- Main Sequence
  - Hydrogen fused in star's core
  - Most of a star's lifetime
  - $0.1 R_\odot \rightarrow 10 R_\odot$

- Red giants
  - H in core depleted, He fusion begins
  - Star expands and outer layer cools
  - $10 R_\odot \rightarrow 1000 R_\odot$

- White dwarfs
  - End state of most main sequence stars
  - $0.01 R_\odot \simeq R_E$
  - $T_{wd} \sim 25000\,k \rightarrow 3000\,k$



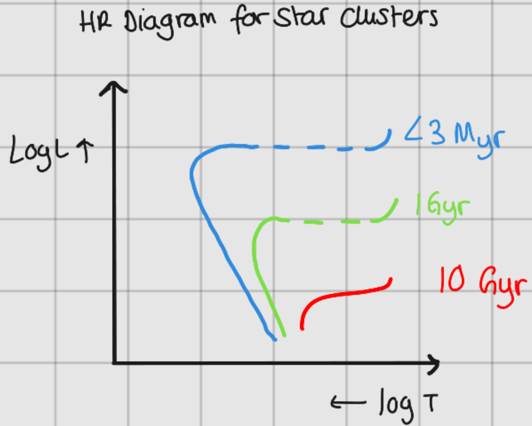
Theoretical HR diagrams give L and T

$$L = 4\pi R^2 \sigma T^4$$

Star Clusters

- Useful to study stars as they contain stars of the same age and composition

Open clusters (100 – 1000 stars)
- Found in disk of our galaxy
- All stars very young (1 mil – 1 bil years)



Globular Clusters ($10^5 - 10^6$ stars
- All over the galaxy
- Very old stars (~12 Gyr)

Physical Conditions

- Most stars are in hydrostatic equilibrium → pressure supports fluid above any point

From $p = \rho g h$, $h = R$ and $g = \frac{GM}{R^2}$ and $\rho = \bar{\rho} = \frac{M}{4/3 \pi R^3}$

$$\boxed{P_c = \rho \frac{GM}{R} = \frac{3}{4\pi} \frac{GM^2}{R^4}}$$

P_c = Central pressure

Assuming $pV = n R_g T$

$$\boxed{p = \frac{n N_A}{V} k_B T = n_d k_B T = \frac{\rho k_B T}{\mu}}$$

n_d = number density of particles

μ = mass per particle

Then using $P_c = \frac{GM\bar{\rho}}{R} = \frac{\bar{\rho} k_B T}{\mu}$ gives $\boxed{k_B T = \frac{GM\mu}{R}}$

Thermal E = gravitational E

Stellar Evolution

- All thermally supported stars must have a finite lifetime as they lose heat

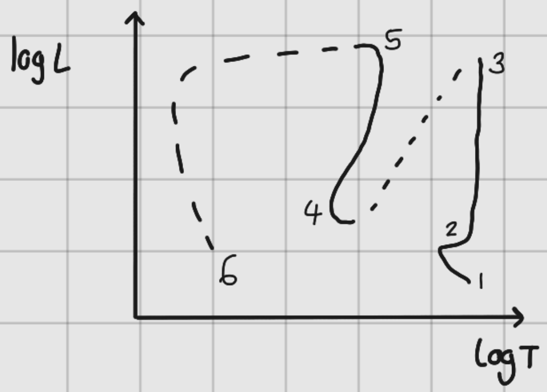
$\boxed{t = \frac{E}{L}}$ t= lifetime of Star
E= total energy available L= luminosity

- The Source of stellar energy must be from hydrogen fusion

- fusion energy supply $\propto$ mass
- $L \propto M^4$

$\boxed{t \propto \frac{1}{M^3}}$

Stellar Evolution



① Main Sequence
- $H \to He$ fusion in core
- Most of Stars lifetime

② Red giant
- H depleted in core → He fusion in shell around core
- Envelope of star expands

③ Horizontal branch phase
- $T \sim 10^8 K$
- He Starts to fuse in core

④ Asymptotic Giant Branch (AGB)
- He depleted in core
- He fusion in shell around core, with H fusion shell around it

⑤ White dwarf

- Final state of star depending on its mass
  - ↳ $M < 0.08 M_\odot$ → Brown dwarfs
    - Don't get hot enough to fuse H
  - ↳ $0.08 M_\odot < M < 0.8 M_\odot$ → Red and k dwarfs
    - H fusion but low L means $t >$ age of universe
  - ↳ $0.8 M_\odot < M < 5 M_\odot$ → White dwarfs
    - H fusion, lifecycle of the sun
  - ↳ $5 M_\odot < M < 25 M_\odot$ → Neutron stars
  - ↳ $M > 25 M_\odot$ → Black holes

Using Pauli's exclusion principle and $\lambda = h/p$, the degeneracy energy

$$\boxed{E_d = \frac{p^2}{2m} = \frac{h^2 n_e^{2/3}}{2m_e}}$$

E_d = degeneracy energy
n_e = electron number density

↑ No T dependence!

- degenerate electrons exert a pressure, which supports white dwarfs

- Degeneracy energy can balance gravity indefinitely, as it is not dependent on temperature
  - degeneracy energy scales faster than gravitational, allowing stars to reach stable radii

Neutron Stars:

- For high masses, $E = pc = hc\,n_e^{1/3}$

- Chandrasekhar discovered degenerate objects have a maximum mass, above which they collapse

$$E_{e^-} \propto \left(\frac{M}{R^3}\right)^{1/3} \propto \frac{M^{1/3}}{R} \qquad E_{grav} \propto \frac{M}{R}$$

- If $M\uparrow$ and $R\downarrow$ star can support itself until momentum becomes relativistic, then $E_d \propto M^{1/3}/R$, so ΔM cannot be balanced by ΔR and star collapses

Black holes:

- objects with $V_{esc} > c$ could escape

$$V_{esc}^2 = \frac{2GM}{R} \qquad \text{So for } V_{esc} = c$$

$$\boxed{R_s = \frac{2GM}{c^2}} \qquad R_s = \text{Schwarzchild radius}$$

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Section 10 - Galaxies

> Galaxy – Gravity bound collection of stars, dust and gas

3 Main types:

- Spiral Galaxies (Milky way)
  - Central region (bulge) and Spiral arms
  - Sites of young star formation → contain hot young stars
- Elliptical galaxies
  - Extremely large
  - Contain older, low-mass stars
- Irregular galaxies
  - Small with undistinct shape

- Orbital velocity in a galaxy appears constant, right out to limit of observations
  - most mass is unseen (dark matter)

Stars on circular orbits balance gravitational with centripetal force

$$\frac{GMm_s}{R^2} = \frac{m_s v^2}{R}$$

m_s = mass of star

M = mass interior to stars orbit

$$\boxed{M = \frac{v^2 R}{G}}$$

- Most galaxies believed to have black holes at their Centre
  - Very high mass

- Galaxies are not randomly distributed in space → major concentrations of galaxies ← galaxy clusters
  ↳ likely due to the way universe has evolved since big bang
    ↳ clumping of dark matter

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Section 11 - The Universe

Hubbles law

- Hubble found v_R of galaxies increased with d (distance to galaxy)
  - always redshifts → Galaxies moving away from us

$$\boxed{v = H_0 d}$$

v = recession speed
H_0 = Hubble constant
d = distance to galaxy

- Hubbles constant is a function of time $H_0 = H(t_0)$ t_0 = age of universe

$t_0 = H_0^{-1} = 13.6\ \text{Gyr}$

- The expansion of the universe can be measured by redshift, z

$$\boxed{1 + z = \frac{\lambda}{\lambda_0}}$$

λ = apparent wavelength
λ_0 = rest wavelength

$z = 0$ is for local objects
$z \sim 0.1$ is for the local universe
$z > 1$ is for the high-redshift universe (distant universe)

$$\boxed{v = cz}$$

v = recession velocity

- Cosmic Microwave background radiation can be detected from all directions in the Sky
  - Marks time about 380000^y after big bang where universe first became transparent
  - Evidence for big bang

- Big bang theory accounts for light elements seen in the universe
  - All other elements up to iron fused in Stars
  - Elements heavier than iron formed in explosions

→ Explains large scale distribution of galaxies
↳ Dark matter needs to be moving slowly

- If the universe is close to a critical density, ρ_c

$$\frac{1}{2} m v^2 = \frac{GMm}{R}$$

M = universe mass
R = universe radius

$$\frac{1}{2}(H_0 R)^2 = \frac{4}{3}\pi R^3 \rho_c \frac{G}{R}$$

$$H_0^2 R^2 = \frac{8}{3}\pi R^2 \rho_c$$

$$\boxed{\rho_c = \frac{3H_0^2}{8\pi G}}$$

- 5% is baryonic matter
- 25% is dark matter
- 70% is unknown (Dark energy