MATHS

$$\oint_S \vec{W}\cdot d\vec{S} = \int_V \vec{\nabla}\cdot W \, dV$$

GAUSS' THEOREM

$$\oint_C \vec{W}\cdot d\vec{l} = \int_S \vec{\nabla}\times\vec{W}\cdot d\vec{S}$$

STOKE'S THEORM.

MAXWELL'S EQUATIONS IN FREE SPACE. (USUALLY COMES UP IN Q1).

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$ continuity equation for electric charge.

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ M1 - from Gauss' law $\left(\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho \, dV\right)$

$$\vec{\nabla}\cdot\vec{B} = 0$$ M2 - Solenoidal Condition. $\left(\oint_S \vec{B}\cdot d\vec{S} = 0\right)$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ M3 - Faraday - Lenz law of Induction. $\left(\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}\right)$

$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \underbrace{\epsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\text{displacement current density}}\right)$$ M4 - AMPERE'S LAW + CONTINUITY.

ELECTROSTATICS. (2020 PAPER Q1)

$$\vec{E} = -\vec{\nabla}\psi \qquad \nabla^2\psi = -\rho/\epsilon_0 \qquad \nabla^2\psi = 0.$$

Poisson's eqn. Laplace's eqn ($\rho = 0$).

ELECTROMAGNETIC WAVES IN FREE SPACE.

Take curl of M3, sub in M4 for $\vec{\nabla}\times\vec{B}$ to derive:

use the identity $\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = \vec{\nabla}\times(\vec{\nabla}\times\vec{E})$

$$\nabla^2\vec{E} = \mu_0\epsilon_0 \frac{\partial^2\vec{E}}{\partial t^2} \qquad \nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}. \qquad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

~~in~~ with $E = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ using this on M3 and M4:

$$\vec{\nabla} \to i\vec{k} \qquad \frac{\partial}{\partial t} \to -i\omega$$

$$\frac{E}{B} = \frac{\omega}{k} = c.$$

MAXWELL'S EQUATIONS IN MATTER.

$\sigma_p = \vec{P}\cdot\hat{n}$ $\quad\quad$ $\rho_p = -\vec{\nabla}\cdot\vec{P}$ $\quad\quad$ $\vec{J}_p = \dfrac{\partial\vec{P}}{\partial t}$. $\quad\quad$ $\vec{P} = \epsilon_0 \chi \vec{E}$.

σ_p ↓ Surface charge density

ρ_p ↓ volume density

$\vec{J}_p$ ↓ polarisation current density

χ ↓ susceptibility.

Polarisation is the electric dipole moment per unit volume.

ISOTROPIC : No direction is special, response of the material is parallel to E-field in all directions.

LINEAR: Response is proportional to applied field. χ is a number.

$\vec{j}_M = \vec{M}\times\hat{n}$ $\quad\quad$ $\vec{J}_m = \vec{\nabla}\times\vec{M}$

$\vec{j}_M$ ↓ surface current due to $\vec{M}$.

↳ volume magnetisation current.

Magnetisation is the ~~[illegible]~~ magnetic moment per volume. $\quad\quad$ $(\vec{m} = I\vec{A})$.

$\vec{\nabla}\cdot\vec{D} = \rho_f$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

$\vec{\nabla}\times\vec{H} = J_f + \dfrac{\partial\vec{D}}{\partial t}$.

$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_0\underbrace{(1+\chi)}_{\text{from above.}}\vec{E} = \epsilon_0\epsilon_r\vec{E}$

$\epsilon_r = 1+\chi$.

* consider gauss' law with free charges + bound polarisation charges.

$\rho = \rho_f - \vec{\nabla}\cdot\vec{P}$.

$\vec{B} = \mu_0(\vec{H}+\vec{M})$

* consider $J = J_F + J_M + J_P$ and sub into M4 - Ampere's law.

$(\vec{B} = \mu_0\mu_r\vec{H})$. $(M = \chi_m\vec{H}$, $\vec{B} = \mu_0(1+\chi_m)\vec{H})$.

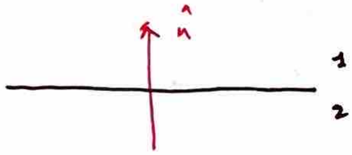
BOUNDARY CONDITIONS.

$\hat{n}\times(\vec{E}_2 - \vec{E}_1) = 0$ $\quad\quad$ $E_{2,\parallel} = E_{1,\parallel}$

$\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \sigma_p$ $\quad\quad$ $D_{1,\perp} - D_{2,\perp} = \sigma_f$

$\hat{n}\cdot(\vec{B}_1 - \vec{B}_2) = 0$ $\quad\quad$ $B_{1,\perp} = B_{2,\perp}$.

$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{j}_f$ → parallel to boundary.



$\vec{J} = g\vec{E}$ $\quad\quad$ Ohm's Law.

↳ conductivity.

WAVES AND ENERGY DENSITY.

Using Maxwell's eqns. with $J_f = 0$, $\rho_f = 0$.

Can derive EM wave equations as before. (Take curl of M3, use identity, sub in M4)

↪ or the other way round for magnetic field.

$\nabla \times$ M4 - identity - sub in M3.

$$v_\phi = \frac{c}{\sqrt{\mu_r \epsilon_r}} \qquad n = \sqrt{\mu_r \epsilon_r}.$$

using $\vec{\nabla} \to i\vec{k}$ and $\partial/\partial t \to -i\omega$ here gives;

$\vec{k} \cdot \vec{D} = 0$

$\vec{k} \cdot \vec{B} = 0$

$\vec{k} \times \vec{E} = \omega \vec{B}$

$\vec{k} \times \vec{H} = -\omega \vec{D}$

defining Impedence:

$$\frac{E}{H} = Z$$

$$\left(\frac{E}{B} = \frac{\omega}{k} = v_\phi = \frac{c}{n}\right).$$

a few relations.

$$\frac{E}{H} = \mu_0 \mu_r \frac{\omega}{k} = Z, \quad \frac{\omega}{k} = \frac{Z}{\mu}.$$

$$Z = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \quad \text{or} \quad Z = Z_0 \sqrt{\frac{\mu_r}{\epsilon_r}}, \quad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.73\ \Omega.$$

ENERGY DENSITY + FLUX.

$$u = \frac{1}{2}\vec{E} \cdot \vec{D} + \frac{1}{2}\vec{H} \cdot \vec{B}$$

$$\left(\vec{E} \cdot \vec{D} = \epsilon E^2 \qquad \vec{B} \cdot \vec{H} = \mu H^2 = \mu\left(\frac{E}{Z}\right)^2 = \epsilon E^2 \right).$$

$\therefore u = \vec{E} \cdot \vec{D}$ because $\frac{\mu}{Z^2} = \epsilon$

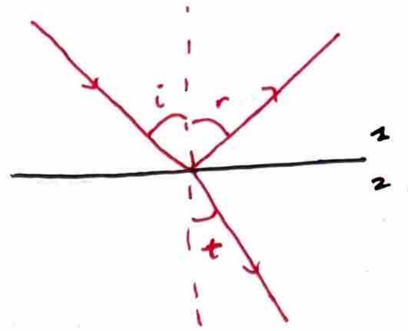
$$\vec{S} = \vec{E} \times \vec{H}$$

↪ Flux - Poynting vector. - might need to find time averages in questions.

* $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$ and $\langle \cos^2(\omega t) \rangle = \frac{1}{2}$.

EM WAVES IN MATTER WITH BOUNDARIES.

Need to match at boundary - at all places, for all time.



$\therefore \omega_i = \omega_r = \omega_t$.

and parallel to interface.

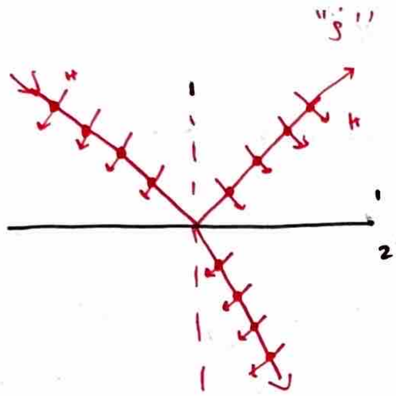
$k_i \sin i = k_r \sin r = k_t \sin t$

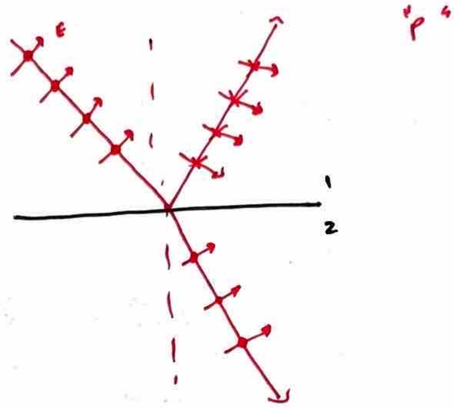
as $i = r$ and $\frac{\omega}{k} = \frac{c}{n}$, with same ω and c, $k \propto n$.

$i = r$

$n_1 \sin i = n_2 \sin t$. Snell's law.

FRESNEL RELATIONS.





Matching components parallel of $\vec{E}$ and $\vec{H}$, give Fresnel relations.

$\tan i_B = \frac{n_2}{n_1}$

r_p goes from negative to positive.
is 0 at Brewster's angle.

SKIN EFFECT.

$\delta = \sqrt{\frac{2}{\mu g \omega}}$

use $\vec{J}_f = g\vec{E}$ in Maxwell's equations.
for $g \gg \epsilon\omega$ - good conductor.

- OPTICS -

- Fermat's principle: optical length is extremised.

- SPHERICAL INTERFACE, use $y^2 = (2R - x)x$ – intersecting chords theorem, find τ, maximise it $\frac{d\tau}{dx} = 0$

"Real is Positive".

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}.$$

THIN LENS.

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

In Cartesian.

$$-\frac{1}{u'} + \frac{1}{v'} = \frac{1}{f}.$$

for telescopes magnification $\frac{D_1}{D_2} = \frac{f_1}{f_2}$

$$M = \frac{f_1}{f_2}$$

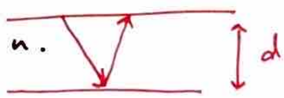
$$\boxed{M = \theta / \alpha}.$$

WAVE OPTICS.

$\Delta\theta = 1.22\frac{\lambda}{D}$. → Rayleigh.

THIN FILM INTERFERENCE.

~~if~~ Phase shift → $2k_z d$



$2\left(\frac{2\pi}{\lambda_4}\right)d \rightarrow \lambda = \frac{\lambda_0}{n}$

$\therefore$ to get, $\Delta\phi = \pi$, $d = \frac{\lambda}{4}$.