PX156 Quantum Phenomena and Particle Physics

Part A – Quantum

1- Blackbody Radiation

> Blackbody – an ideal absorber that absorbs all radiation that strikes it
  – also an ideal emitter

> Blackbody radiation – continuous spectrum radiation emitted by an ideal blackbody

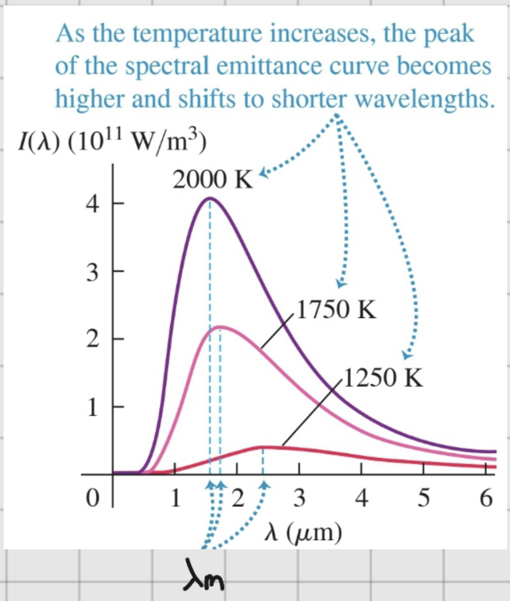
$$\boxed{I = \sigma T^4}$$

→ Stefan-Boltzmann law
σ = Stefan-Boltzmann constant

↳ This is not uniformly distributed over all wavelengths
– useful to use <u>Spectral emmittance</u> – $I(\lambda)$ where total intensity

$$\boxed{I = \int_0^\infty I(\lambda)\, d\lambda}$$

> Spectral emmittance – power per unit wavelength interval
– Units Wm^{-3}
↳ not the same thing as Intensity



Wien's displacement law:

$$\boxed{\lambda_{max} T = 2.898 \times 10^{-3}\ mK}$$

$$\boxed{I(\lambda) = \frac{2\pi c k_B T}{\lambda^4}}$$

- The Rayleigh-Jeans law was not derived experimentally

- Agrees with experimental results at high wavelengths
- As $\lambda \to 0$, $I(\lambda) \to \infty$, not 0 as experimentally found
  ↳ the ultraviolet catastrophe

Planck derived a model for BB radiation:

- Energy quantised to $E_n = nhf \quad n \in \mathbb{Z}$

$$\boxed{\langle E \rangle = \frac{hf}{\exp(hf/k_B T) - 1}}$$

$$\boxed{I(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}}$$

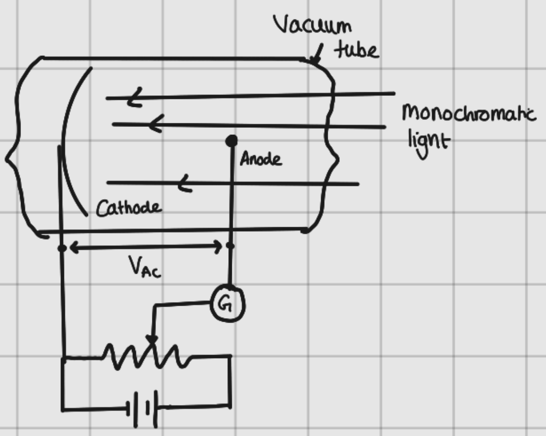
→ planck's radiation law

2 - The Photoelectric Effect

$\boxed{E = hf}$ → photon energy

$\boxed{E_{kmax} = hf - \varphi}$ → Photoelectron maximum kinetic energy

> Work function, φ – minimum energy required to remove electrons from a metals surface



<u>Experimental observations</u>

- Electrons emitted with range of KE
- Maximum KE dependent on light frequency and independent of light intensity
- No electrons emitted if frequency is below a threshold value

Considering a photon as a relativistic particle:

$$E_T = KE + m_0c^2$$
$$= (\gamma - 1)\, m_0c^2 + m_0c^2 = \gamma m_0 c^2 = \frac{m_0c^2}{\sqrt{1 - \frac{u^2}{c^2}}}$$

as $u \to c$, $E \to \infty$ if $m_0 \neq 0$, but $E = hf$ so $m_0 = 0$

As $m_0 = 0$, $E = pc$ $\rightarrow$ $\boxed{p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}}$

3- Compton Scattering

- A beam of xrays is fired at a solid target and the wavelength of scattered radiation is measured

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$

λ' - wavelength of scattered radiation
λ - wavelength of incident radiation
θ = Scattering angle

Example:

A photon scatters off a stationary electron. If the photon emerges at an angle of 60° to its initial direction and its wavelength of scattering is doubled, calculate the angle at which the electron recoils, ϕ:

$\theta = 60°$ $\lambda' = 2\lambda$ $p' = \frac{p}{2}$ ← from $p = \frac{h}{\lambda}$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$2\lambda - \lambda = \lambda_c(1 - \cos 60°)$$

$$\lambda = \frac{h}{2m_e} = \frac{1}{2}\lambda_c$$

horizontal: $p = p' \cos\theta + p_e \cos\phi = \frac{p}{2}\cos\theta + p_e \cos\phi$

vertical: $p' \sin\theta = \frac{p}{2}\sin\theta = p_e \sin\phi$

$$\frac{p_e \cos\phi}{p_e \sin\phi} = \cot\phi = \frac{p\left(1 - \frac{1}{2}\cos\theta\right)}{p/2 \sin\theta} = \frac{2 - \cos\theta}{\sin\theta}$$

For $\theta = 60°$: $\tan\phi = \frac{\sin 60°}{2 - \cos 60°} = \frac{1}{\sqrt{3}}$ $\quad \phi = 30°$

4 - Atomic Spectra

Bohr's model of the atom:

- Electrons orbit the nucleus at discrete distances r_n

$$\boxed{r_n = \frac{n^2 \varepsilon_0 h^2}{\pi m_e e^2}}$$

- Electrons lose/gain energy by emitting/absorbing photons, which allows them to move between energy levels

The angular momentum of electrons, L, is limited by

$$\boxed{L = n\hbar}$$

The total energy for the nth orbit of hydrogen is

$$\boxed{E_n = -\frac{hcR}{n^2} = -\frac{13.6\,eV}{n^2}}$$

The wavelength of a photon emitted in a transition from state n_1 to n_2

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)}$$

5- de Broglie Wavelength

- The de Broglie wavelength of a particle is

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

- Bragg Scattering is the diffraction of x-rays in atomic spacing

$$\boxed{n\lambda = 2d\sin\theta}$$

6- Schrödinger's Equation

For a free particle of mass m:

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t}}$$

Where Ψ is the wave function, representing the probability of finding a particle at a point in space at a certain time

↳ A plane wave $\Psi = e^{ikx - i\omega t}$ Satisfies this

The relationship between ω and k can be derived to be

$$\boxed{\hbar\omega = \frac{\hbar^2 k^2}{2m}}$$

The probability density function can be introduced:

$$\boxed{|\psi(x,t)|^2 = \psi(x,t)\,\psi^*(x,t)}$$

The normalisation function to give the probability of finding the particle in all given locations in the considered space

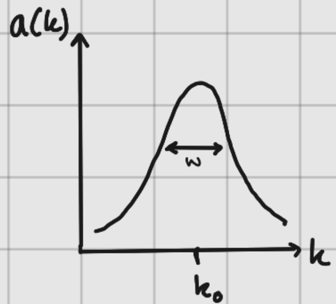
$$\boxed{\int |\psi(x,t)|^2 \, dx = 1}$$

> Wave packet – Superposition of many plane waves with an average wavenumber of k_0
  - ∟ by superposing many waves, one wave is constructed with one maximum of $|\psi(x,t)|^2$
    - This looks like a particle in that it is localised in space

A wave packet can be represented as

$$\boxed{\psi = \int a(k)\, e^{ikx - i\omega(k)t} \, dk}$$

where $a(k)$ is k-dependent amplitude



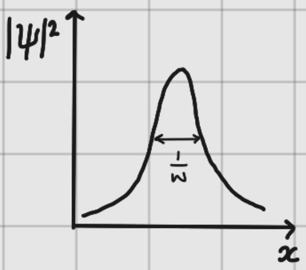
w is the full width at half maximum

$$\boxed{w = 2\sqrt{\ln 2}\,\sigma}$$

Where the distribution is narrow around k_0, ψ is given by

$$\boxed{\psi = \exp(ik_0x - i\omega_0 t)\underbrace{\int a(k)\exp(i\Delta k(x-\omega_0' t))\,dk}_{f(s)\,=\,f(x-\omega_0' t)}}$$

which represents a travelling wave with $V_g = \omega_0$



For a wave with width w in k, a top-hat distribution can be used

$$a(k)\begin{cases} = A & k_0 - \frac{w}{2} < k < k_0 + \frac{w}{2} \\ = 0 & \text{otherwise}\end{cases}$$

Time Independent Schrodinger Equation

If a particle is not free, it has a potential energy $V(x)$, and Schrödinger's equation becomes

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

by letting $\psi(x,t) = \phi(x)\rho(t)$

$$\rightarrow i\hbar\frac{\partial}{\partial t}\left(\phi(x)\rho(t)\right) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\phi(x)\rho(t)\right) + V(x)\phi(x)\rho(t)$$

$$i\hbar\,\phi(x)\frac{d}{dt}\rho(t) = -\frac{\hbar^2}{2m}\rho(t)\frac{d^2\phi(x)}{dx^2} + V(x)\,\phi(x)\,\rho(t)$$

Dividing by $\phi(x)\rho(t)$

$$\frac{i\hbar}{\rho}\frac{d\rho}{dt} = \frac{-\hbar^2}{2m\phi}\frac{d^2\phi}{dx^2} + V(x) = E$$ where E is constant

$i\hbar\frac{d\rho}{dt} = E\rho$ has solution $\rho(t) = \exp(-iEt/\hbar)$

$$\boxed{\frac{-\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V\phi = E\phi}$$

→ Solution depends on precise form of $V(x)$

The probability density of the TISE is given by:

$$|\psi|^2 = \phi\phi^*\rho\rho^*$$

- this is independent of time as ρ is essentially a phase term
- This is a Stationary State

7 - Heisenberg's Uncertainty Principle

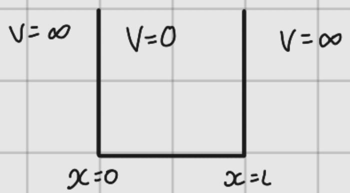
$$\boxed{\Delta x\,\Delta p \geq \frac{\hbar}{2}} \qquad \boxed{\Delta x = \langle x^2\rangle - \langle x\rangle^2}$$

It is impossible to simultaneously determine both the position and momentum of a particle with arbitrarily great position

8 - Potential Wells

> Potential well - The region surrounding a local minimum of potential energy

Infinite Potential Well



$V(x) = \infty$ for $x<0$ and $x>L$
$V(x) = 0$ for $0 \leq x \leq L$

So that the particle cannot exist outside the well with infinite energy
$\phi(x) = 0$ for $x<0$ and $x>L$

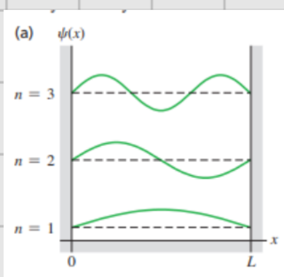
For $0 \leq x \leq L$ $V(x) = 0$ So $\frac{-\hbar^2}{2m}\frac{d^2\phi}{dx^2} = E\phi$

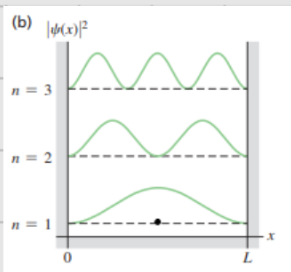
↳ $\frac{d^2\phi}{dx^2} + \frac{2mE}{\hbar^2}\phi = 0$

↳ $\frac{d^2\phi}{dx^2} + k^2\phi = 0$ where $k^2 = \frac{2mE}{\hbar^2}$

which has solution $\phi = A\sin(kx) + B\cos(kx)$

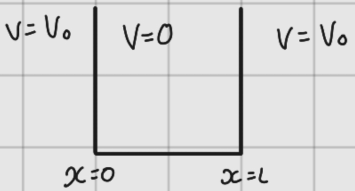
- Using boundary conditions $\phi(0) = 0$ and $\phi(L) = 0$, $B = 0$, $k = \frac{n\pi}{L}$
  So $\phi(x) = A\sin(n\pi x/L)$
- Use normalisation for $\int_0^L (A\sin(n\pi x/L))^2 dx = 1$ to find $A = \sqrt{\frac{2}{L}}$





The value of $|\psi(x)|^2 dx$ at each point represents the probability of finding the particle in small interval dx about the point

Finite Potential well



$V(x) = V_0$ for $x<0$ and $x>L$

$V(x) = 0$ for $0 \leq x \leq L$

TISE: $\dfrac{-\hbar^2}{2m}\dfrac{d^2\phi}{dx^2} + V_0\phi = E\phi$

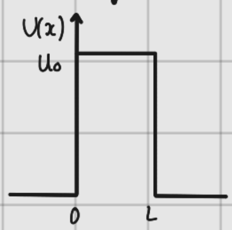
- Inside the well $\dfrac{d^2\phi}{dx^2} + k^2\phi = 0$ $\qquad k^2 = \dfrac{2mE}{\hbar^2}$

- Outside the well $\dfrac{d^2\phi}{dx^2} - \dfrac{2m}{\hbar^2}(V_0 - E)\phi = 0$ ← Cannot be solved analytically

9 - Potential Barrier

> Potential barrier - A potential energy function with a local maximum

- In classical mechanics, if a particle has total mechanical energy less than the the maximum energy of the barrier, it cannot pass through
- In quantum mechanics, the particle may pass through → tunelling



→ The functions in and out of the barrier must join smoothly. So $\psi(x)$ and $\dfrac{d\psi}{dx}$ must be continuous at $x = 0, L$

- Before the barrier $\frac{d^2\phi_1}{dx^2} + \frac{2mE}{\hbar^2}\phi_1 = 0$ with $\phi_1 = Ae^{ikx} + Be^{-ikx}$
- After the barrier $\frac{d^2\phi_3}{dx^2} + \frac{2mE}{\hbar^2}\phi_3 = 0$ with $\phi_3 = Fe^{ikx} + Ge^{-ikx}$

- Inside the barrier $\frac{d^2\phi_2}{dx^2} + \frac{2m}{\hbar^2}(E - U_0)\phi_2 = 0$ with $\phi_2 = Ce^{ik'x} + De^{-ik'x}$

where $k^2 = \frac{2mE}{\hbar^2}$ and $k' = \frac{\sqrt{2m(E-U_0)}}{\hbar}$

At boundary conditions:

$x = 0$: $\phi_1 = \phi_2$ $\frac{d\phi_1}{dx} = \frac{d\phi_2}{dx}$

$x = L$: $\phi_3 = \phi_2$ $\frac{d\phi_3}{dx} = \frac{d\phi_2}{dx}$

The transmission probability is given by

$$\boxed{T \propto e^{2k'L}}$$

10- Alpha Decay

$$\boxed{\frac{dN}{dt} = -\lambda N}$$

$$\boxed{N(t) = N(0)e^{-\lambda t}}$$

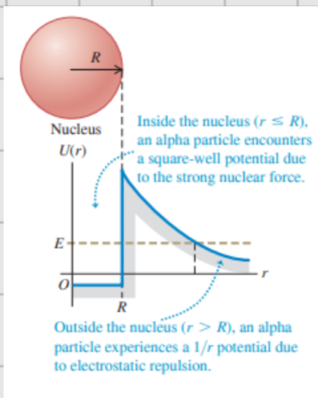
N - number of nuclei remaining

λ - decay rate - probability per second

$$\boxed{\tau = \frac{1}{\lambda}} \qquad \boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

→ $t_{1/2}$ = half life
→ τ = particle lifetime

In α decay, the α particle tunnels through a potential barrier



→ If an α inside the nucleus has $E>0$, it can escape the nucleus

PARTICLES:

Bosons

- Force associated/carriers
- $n\hbar$ Spin $n \in \mathbb{Z}$
  ($n=0$ for Higgs)

Hadrons

- Particles that experience SNF (comprised of quarks)
- 0 net colour charge and integer electric charge

Fermions

- Matter particles
- Spin $\frac{n\hbar}{2}$ for odd n
- 2 classes, each with 3 generations, quarks and leptons

Quantum numbers: Baryon, L_e L_μ L_τ, electric charge, weak isospin, flavour #, energy*, momentum*, Colour, quark flavour

▬ = Conserved by all forces

Quark	Flavor #
u	+1/2
d	-1/2
c, t	+1
s, b	-1

Particle	T_W
u, c, t	+1/2
d, s, b	-1/2
e, μ, τ	-1/2
ν_e, ν_μ, ν_τ	+1/2

opposite for (right handed) antiparticles

* $m_f \leq m_i$

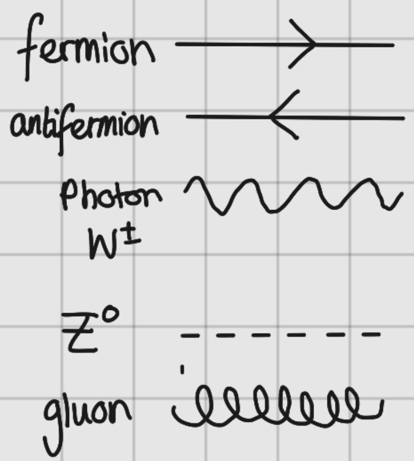
Weak isospin:

- $W^{\pm}$ Carries $T_W = \pm 1$, so interactions change T_W of lepton by 1

Colour:

- quarks have colour charge
- r, g, b → $r\bar{r}$ = Colourless rgb = Colourless

FEYNMANN DIAGRAMS



- gluons only emitted by quarks + gluons
- photons emitted / absorbed by leptons + quarks
- Z^0 emitted / absorbed by everything except gluons

- W transforms within lepton generations
- W transforms cross-generationally for quarks, but only changing charge
- No flavour changing neutral interactions

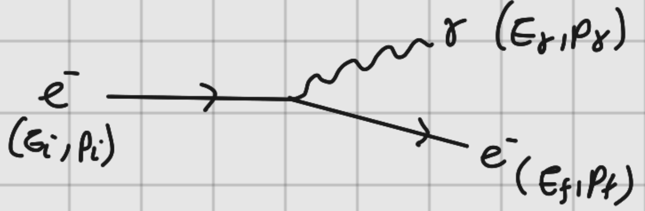
Hierarchy of force choice: Strong → EM → Weak (fastest → slowest)

Crossing Symmetry: Particles can move from $i \leftrightarrow f$ by turning into antiparticles

↳ $u \to W^+ + d$ or $u + W^- \to d$ or $W^- \to d + \bar{u}$ ...

Changes:
- $Z^0/\gamma/g \leftrightarrow q_1\bar{q}_1$
- $Z^0 \leftrightarrow \nu_1\bar{\nu}_1$
- $Z^0/\gamma \leftrightarrow l_1^+ l_1^-$
- $W^\pm \leftrightarrow q_1\bar{q}_2$
- $W^\pm \leftrightarrow l\,\nu_l$
- Net flavour change → $W^\pm$

Showing Virtual particles have no mass:



- E_i and p_i fixed, so 4 degrees of freedom
- $E_i = (p_i c)^2 + (m_e c^2)^2$ so 3 dof
- $E_i = E_i + E_\gamma$ so 2 dof
- $p_i = p_f + p_\gamma$ so 1 dof
- $E_f = (p_f c)^2 + (m_e c^2)^2$ so 0 dof

↳ $E_\gamma = p_\gamma c \to m_\gamma = 0$

Trying 2 separate 2 quarks
- Separation ↑ = energy density in gluon field ↑
- A some point E is enough to create 2 new quarks

FORCES:

Strong

- Acts on particles with colour charge
- Exchange particle is gluon
  - $m, q, T_W = \emptyset$, Colour $= C_1 \overline{C_2}$
- Weak at short distances, strong at large
  - Short r, $F \sim 1/r^2$ like EM
  - long r, $F \sim -k$ $(k = 1\ \text{GeV/fm})$
- Fastest $\sim 10^{-20}$ s $\gg$

- only force that conserves colour
- conserves quark flavor

Weak

- Acts on particles with weak isospin
- Exchange particle is $W^{\pm}$ for charged, Z^0 for neutral
  - W can couple to W or Z^0
- Responsible for decay
- Short range: $F = \alpha_W \frac{e^{-r/R}}{r^2}$
- Slowest $\sim 10^2 - 10^{-9}$ s

- doesn't conserve quark flavor

Charged Current

- Carried by $W^{\pm}$ boson
  - $W^{\pm}$: $T_W, q = \pm 1$
  - heavy
  - only boson to change flavour of $q + \ell$

Neutral Current

- Carried by Z^0
  - Z^0: $T_W, q = \emptyset$
  - heavier than $W^{\pm}$

→ Only couples particles with their $\bar{p}$
↳ No FCNC

Electromagnetism

- Exchange particle is photon, γ
  - No m, q; doesn't interact with γ
- Acts between electrically charged particles
- All quantum numbers conserved
- Infinite range $F \sim 1/r^2$
- Second fastest $\sim 10^{-15}$ s

- conserves all quantum numbers

Gravity

- Exchange particle is Higgs boson

SYMMETRY

Continuous:

> Symmetries involving a continuous change in some parameter defining the geometry of the system

* see Noether's theorem

Translation in			Conserved quantity:
• space	$\vec{x} \to \vec{x} + \vec{k}$	$\to$	linear momentum
• angle	$\theta \to \theta + \theta_0$	$\to$	angular momentum
• time	$t \to t + t_0$	$\to$	energy

↳ laws of physics symmetric under space-time symmetry

Internal:

> An operation on the mathematics describing a system

Gauge Symmetry		Conservation of:
• Wavefunction phase	$\to$	electric charge
• Upness & Downness	$\to$	electroweak charge
• Colour	$\to$	colour

$|\psi|^2 = \psi\psi^*$

If $\psi' = e^{i\alpha}\psi$:

$|\psi'|^2 = \psi'\psi'^* = (e^{i\alpha}\psi)(e^{-i\alpha}\psi^*)$

$= \psi\psi^* = |\psi|^2$

Discrete:

> Symmetries based on non-continuous changes to the system

- Parity, $\hat{P}$ – Spatial inversion $(x \to -x)$ ← $L = \vec{r} \times \vec{p}$ but after $\hat{P}$, $\vec{r} = -\vec{r}$ $\vec{p} = -\vec{p}$ so $L' = -\vec{r} \times -\vec{p} = L$
- Charge Conjugation, $\hat{C}$ - Quantum sign inversion $(p \to \bar{p})$
- Time reversal, $\hat{T}$ – time inversion $(t \to -t)$
- CP $(x \to -x,\ p \to \bar{p})$
- CPT $(x \to -x,\ p \to \bar{p},\ t \to -t)$

↳ All are conserved by strong & EM, but only CPT by weak too

DECAYS

Fermi's Golden Rule: $P(i \to f) \propto (\text{Amplitude}) \times (\text{Phase Space})$

> phase space - tells you how many ways you can distribute energy from the initial state into KE once the final state masses are taken into account

$\boxed{N(t) = N(0)\, e^{-t/\tau}}$ $\boxed{\frac{dN(t)}{dt} = -N(t)\Gamma}$

τ = particle lifetime
$\Gamma = \frac{1}{\tau}$ = decay rate

- Particles have a number of modes to decay. Each mode i has its own decay rate Γ_i

$\boxed{\Gamma_T = \sum \Gamma_i}$ $\boxed{BR_i = \frac{\Gamma_i}{\Gamma_T}}$ BR_i = Branching ratio of a given decay

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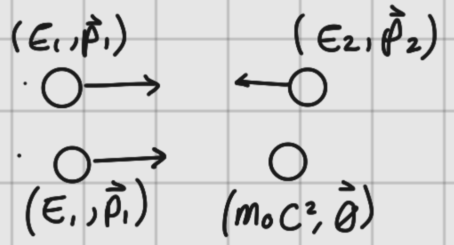
COLLISIONS

- Rest mass is same in all frames

Frame:
- Centre of momentum: $\vec{p}_1 = -\vec{p}_2$
- Fixed target: $\vec{p}_2 = \emptyset$
- Lab frame: Lab is at rest



- s (Mandelstam-s) is invariant for collisions

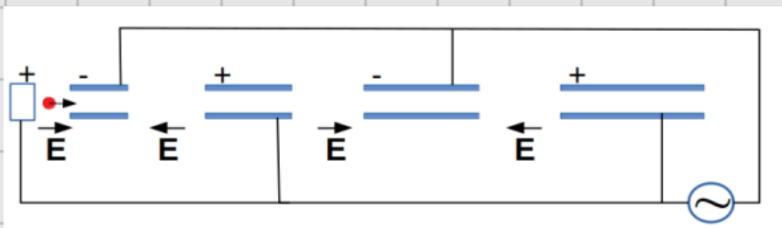
$\boxed{s = (E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2 c^2}$ $\to$ for

↳ In CoM frame ($\vec{p}_1 = -\vec{p}_2$): $s_{com} = (E_1 + E_2)^2$
- $\sqrt{s_{com}}$ = total energy available for particle creation

↳ In fixed target frame ($\vec{p}_2 = \vec{0}$) $s_{fixed} = 2E_1 m_2 c^2 + m_1^2 c^4 + m_2^2 c^4$

ACCELERATORS

LINACS (linear accelerators)



particles accelerated through drift tubes of increasing length with an electric field between them

Cyclotron

$$\boxed{r = \frac{mv}{qB}}$$

- Particle accelerated across gap between 2 electrodes
- Only for non relativistic particles

$$\boxed{f_{cyc} = \frac{qB}{2\pi m}}$$

Synchrotron (eg. LHC)

- particles accelerated with fixed radius but increasing magnetic field
- Particles lose energy as photons (braking radiation) $\Delta E \sim \left(\frac{E}{m}\right)^4 \frac{1}{r}$
  - ↳ $\uparrow r = \downarrow E$ loss , $\uparrow m = \downarrow E$ loss

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DETECTORS

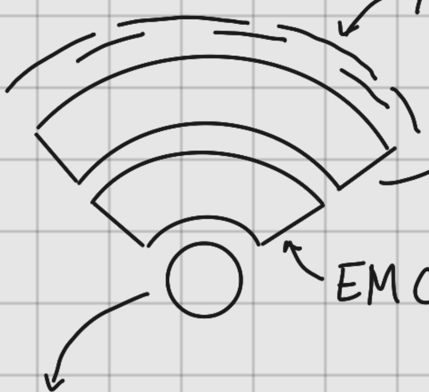
- μ deposit an E_{min} as they move through detector
- e^- and γ initiate EM shower when undergoing EM interaction
- Hadrons initiate hadronic showers when involved in strong interaction

X_0 = radiation length

Probability of e^-/γ passing x through a material unaffected: $\boxed{P(x) = e^{-x/X_0}}$

Probability of Hadron passing x through a material unaffected: $\boxed{P(x) = e^{-x/\lambda_I}}$

Collider Detectors:



μ detector: only μ able to reach this for undetected

Hadronic Calorimeter: Induces H showers
- Thicker than EM calorimeter

EM Calorimeter: Dense material stops e^-/γ and induces EM showers

Tracker: Measures trajectory of colliding charged particles
- in a magnetic field

	Tracker	ECal	HCal	μ chamber
μ	→	→	→	→
$e^{\pm}$	→	⊗		
γ		⊗		
$H^{\pm}$	→	→	⊗	
H^{0}			⊗	
ν				

PARTICLE SOURCES

- β^- decay: $n \to p + e^- + \bar{\nu}_e$
- Cosmic ray showers: Particles from astrophysical sources interact with atoms in atmosphere producing hadronic showers (π^0, k^0 typically, then $\pi^{\pm} \to \mu^{\pm} + \nu$)
- Solar particles
  - Solar neutrinos from fusion in Sun
  - Solar wind: plasma of charged particles emitted from Solar corona

HELICITY

Helicity:

> helicity, h – the projection of spin vector onto the direction of motion of a particle

$$h = \vec{S} \cdot \hat{p}$$

→ $\vec{S}$ doesn't change under $\hat{P}$

- If $h>0$, right-handed helicity
- If $h<0$, left-handed helicity

→ Weak interaction only creates LH p and RH $\bar{p}$

– Weak interaction violates $\hat{C}\hat{P}$ — don't know why

→ there is a fundamental difference between phyics of matter and antimatter

DEFINITIONS

> Spin- quantised intrinsic angular momentum
> Weak isospin (unrelated above) - a quantum number relating to the electrically charged part of the weak interaction
> Force - an exchange of quantum numbers by bosons
> Principle of confinement - quarks can not be separated/isolated
> hadronization - Separating 2 quarks until they create 2 more
> Virtual particle - intermediate particles allowed to have any mass to make kinematics work
> Symmetry - an operation on a system or the mathematics describing it that leaves the system or physics unchanged
> Noether's theorem - Any system which displays continuous, differentiable symmetry embodies an associated conservation law
> axial vectors - vectors invariant to parity transforms

3 Principles of PP: Symmetry, Composition, unification

EQUATIONS

$E^2 = (pc)^2 + (mc^2)^2$ $\quad E = hf \quad \lambda = \frac{h}{p} \quad$ helicity $h = \vec{S} \cdot \hat{p}$

$N(t) = N(0)\, e^{-t/\tau}$

$S = (E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2 c^2$

$\Gamma = \frac{1}{\tau} \quad BR_i = \frac{\Gamma_i}{\Gamma_{Total}}$

$S_{com} = (E_1 + E_2)^2$

$S_{fixed} = 2E_1 m_2 c^2 + m_1^2 c^4 + m_2^2 c^4$

$r = \frac{mv}{Bq} \quad f_{cyc} = \frac{qB}{2\pi r m} \quad \Delta E_{Br} \sim \left(\frac{E}{m}\right)^4 \frac{1}{r}$

$P_{e/r}(x) = e^{-x/x_0} \quad P_H(x) = e^{-x/\lambda_I}$